Gains from Commitment: The Case for Pegging the Exchange Rate

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Abstract

What is the impact of the exchange rate regime on inflation and economic activity? This paper argues that there are substantial benefits to move towards a fixed exchange rate regime. First, we propose a calibrated small open economy model that features an inflationary bias arising from the lack of commitment of central banks conducting discretionary monetary policy when the exchange rate is flexible. In this scenario, pegging the exchange rate to a stable anchor adds credibility to the pegging country and has an impact on both inflation and economic growth. In line with our model, data suggest that after pegging the currency to a stronger anchor currency, such economies benefit from persistently lower inflation rates and higher economic growth.

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1 Introduction

Countries face a choice between managing their own currency in a float exchange rate regime or pegging it to a stronger anchor currency. On the one hand, by floating their currency, countries enjoy the flexibility to counteract shocks to their economy by making use of their autonomous monetary policy. However, by repeatedly acting under discretion, such actions might cause an inflationary bias. On the other hand, when countries fix their currency against a stronger anchor currency, the monetary policy autonomy loss might be compensated by the gain in credibility from commitment that can potentially solve the inflationary bias. Indeed, one narrative for the formation of the Eurozone was the hope of permanently lower inflation in the currency union for all countries (Lane, 2006).

Despite the topical interest in this research theme, the literature still lacks a thorough quantitative and empirical analysis on how important the credibility channel is in accounting for the economic effects of changing the exchange rate regime. In this paper, we aim to fill this gap by highlighting the credibility channel in an estimated quantitative model from which we derive several implications, which then are tested in our empirical analysis on the economic impact of adopting a fixed (or floating) exchange rate regime.

In this study, we revisit the classical question on whether and how the exchange rate regime matters for countries’ economic performance. Our analysis proceeds in 3 steps. First, we build upon the Chari et al. (2020a) model that features several monetary regimes under commitment and discretion and add a time-varying and country-specific credibility parameter for the central bank, which we use to derive three testable implications about the economic behavior of countries that switch their exchange rate regime. Second, with this framework in mind, we calibrate our model using data for Germany and Italy from 1950 to 2017 and show that our model replicates very well the time series for inflation and GDP in both countries. Finally, we test our model implications using data from 169 economies over the last 70 years. In accordance to our quantitative exercise, the data suggest that after pegging the currency to a stronger anchor currency, such economies benefit from persistently lower inflation rates and higher economic growth. On average, we estimate a 15% persistent reduction in inflation rates and a 3% increase in real GDP cumulative growth over 5 years following a pegging episode.

In our theoretical part, we follow the model by Chari et al. (2020a) that features several monetary regimes under commitment and discretion. We add a credibility parameter for the central bank, as in Schaumburg and Tambalotti (2007) and make this parameter time-varying and country-specific. We then derive three testable implications about the behavior of inflation when countries switch their exchange rate regime: First, inflation goes down permanently
when a client country pegs its currency to a more credible anchor country. Second, volatility of inflation goes down permanently as well. This is also true for the former anchor country, when a currency union is formed. Last, GDP should rise when the exchange rate get fixed, as permanently lower inflation also lowers the economic costs from inflation. In a next step, we use data about the behavior of inflation to estimate credibility of two countries of interest: Germany and Italy. The estimated version of the model suggests that Italy’s credibility indeed approached the German level when it pegged the exchange rate to Germany. This has lead to lower and less volatile inflation rates. Furthermore, the estimated credibility parameter suggests that even Germany benefited from currency pegs of other European countries and from the formation of the Eurozone, as Germany’s credibility improved during the pegging events delivering lower and less volatile inflation than before.

In our empirical exercises, we use the most comprehensive dataset available at the country-level, with information on 169 economies over the last 70 years, corresponding to 7,505 country-year observations including 259 pegging episodes and 266 floating episodes identified following Ilzetzki et al. (2019). We start by providing 3 stylized facts on the differences between countries in a float and fixed exchange rate regimes that are in line with the seminal contributions by Bordo and Schwartz (1999); Ghosh et al. (2002); Calvo and Reinhart (2002): 1) inflation is higher and more volatile in floats than in pegs; 2) real GDP growth is higher in pegs; 3) interest rates are higher and more volatile in floats than in pegs. In addition, in the spirit of Eichengreen and Rose (2012), we also performed an event study analysis around changes in exchange rate regimes and confirmed that following a pegging episode countries displayed lower inflation and interest rates and higher economic growth.

Then, to causally test the implications of our model, and after acknowledging that not all changes in the exchange rate regime are unexpected or unrelated to the business cycle of each economy, we use an inverse propensity score weighting methodology to estimate the impact of a change in the exchange rate regime on inflation and economic growth. On average, we estimate a 15% persistent reduction in inflation rates and a 3% increase in real GDP cumulative growth over 5 years following a pegging episode.

By revisiting the classical question on whether and how the exchange rate regime matters for countries’ economic performance, this paper aims at contributing to two strands of literature. On the empirical side, we contribute to the literature that studies the differences between exchange rate regimes and the effect of pegging and floating episodes. In his seminal work, Mussa (1986) showed that the decision to let the exchange rate regime float freely after the Bretton Woods breakdown did not only have an impact on the nominal exchange rate, but also on the real exchange rate. In more recent work, Itskhoki and Mukhin (2019) reconfirm this
finding but emphasize that changes in the exchange rate regime fail to show up in other real macroeconomic variables such as GDP or consumption. Using a sample of the G7 countries excluding Canada plus Spain, they also argue that there is no systematic change of cyclical properties in inflation after a shift of the exchange rate regime. This paper redirects the focus from the cyclical (short-run) properties and the Bretton Woods breakdown episode towards long-run level shifts of macroeconomic variables after different pegging and floating episodes over the last 70 years for 169 countries. We show that inflation and economic growth are persistently affected after an exchange rate regime change. In line with findings from Levy-Yeyati and Sturzenegger (2003); De Grauwe and Schnabl (2008); Ghosh et al. (2014); Harms and Knaze (2021), we find a negative response of inflation and a positive response of economic growth following a pegging episode.

On the theoretical side, the paper relates to the open economy literature that examines the relationship of exchange rate regimes and the economy. We use an estimated version of the Chari et al. (2020a) model. They set up an open economy model and link it to discretionary monetary policy in the Barro and Gordon (1983) tradition. Models in that tradition point to the signaling content of the regime choice. Governments and monetary authorities that suffer from a credibility deficit can signal their commitment to tough policies by appropriately choosing the exchange rate regime Giavazzi and Pagano (1988). Indeed, Obstfeld et al. (2010) show that countries inherit the monetary stance of their corresponding anchor. Such a shift in credibility that we model as in Schaumburg and Tambalotti (2007) is able to mitigate the inflation bias arising from a discretionary monetary authority. Our paper therefore emphasizes gains from commitment by moving towards a pegged exchange rate regime. Other papers that discuss the stability of those exchange rate arrangements focus on trade gains or invoicing complementarities, see Arvai (2021) and Muhkin (2021). The literature that highlights the disconnect from exchange rate regimes and macro fundamentals (originally Meese and Rogoff (1983) and Itskhoki and Mukhin (2019), Corsetti et al. (2021) more recently) focus on short-term real macro fundamentals. Our finding stresses the permanent effect on the level of inflation and the corresponding impact on real variables stemming from such a permanent shift in inflation. This is in line with Froot and Ramadorai (2005) who find that short-term movement of the exchange rate are often disconnected with macro fundamentals while long-term movements indeed show a relationship to fundamentals.

The remainder of this paper is structured as follows. Section 2 introduces the model and derives 3 implications on the economic behavior of countries that move towards a more fixed exchange rate regime. Sections 3 and 4 present our calibration strategy and our quantitative exercise based on Germany and Italy. Section 5 describes the empirical strategy and presents
its results which are then discussed in Section ?? Section 6 concludes.

2 Model

In this section, we first describe the model that follows closely Chari et al. (2020a). We then expand their setup by adding a credibility parameter as in Schaumburg and Tambalotti (2007). The credibility parameter is time-varying and depends on the exchange rate regime. The goal is to use such a model setup to derive testable implications about the behavior of inflation, interest rates and growth under different regimes. We then provide an estimated version of the model and run a simulation under different monetary regimes. Finally, we compare the model outcome with the data.

2.1 Setup

The model closely follows Chari et al. (2020a). The economy consists of a continuum of countries. Each country produces traded and non-traded goods. The traded good sector is assumed to be perfectly competitive while the non-traded good sector has imperfect competition and sticky prices. This assumption reflects the notion that flexible exchange rates are desirable as they ensure that the relative prices of traded goods to non-traded goods move as if all prices were flexible.

There are two different sources of shocks that hit the non-traded sector only: A markup shock and a productivity shock. Each of these shocks can happen on an aggregate level that hits the whole world equally and on a country-specific level. We adopt the same notation as in Chari et al. (2020a) and denote $z_t = (z_{1t}, z_{2t}) \in Z$ as an aggregate shock in time $t$ where the subindex 1 refers to the markup shock and the subindex 2 to the productivity shock. The country-specific shock $v_t = (v_{1t}, v_{2t}) \in V$ is drawn each period. All of the shocks are i.i.d. over time and across country. The probability of aggregate shocks is $f(z_{1t}, z_{2t}) = f^1(z_{1t}) f^2(z_{2t})$, while the probability for country-specific shocks is given by $g(v_{1t}, v_{2t}) = g^1(v_{1t}) g^2(v_{2t})$. Let $s_t = (s_{1t}, s_{2t})$ summarize the current state of the world with $s_{1t} = (z_{1t}, v_{1t})$ and let $h(s_t) = h^1(s_{1t}) h^2(s_{2t})$ denote the probability of that specific state with $h^i(s_{1t}) = f^i(z_{1t}) g^i(v_{1t})$. In particular let $A(s_{2t})$ denote the productivity shock and $\theta(s_{1t})$ denote the markup shocks to the non-traded sector. The conditional mean of the shocks is given by $E_v(\theta | z) = \sum_{v_{1t}} g^1(v_{1t}) \theta(z_{1t}, v_{1t})$ and $E_v(A | z) = \sum_{v_{2t}} g^2(v_{2t}) A(z_{2t}, v_{2t})$. The timing is as in Chari et al. (2020a). First the markup shock is realized, then non-traded good firms set

\footnote{This keeps the model tractable, as it becomes static. There is no persistence such that a large shock today affects future states. The calibration discusses the shock process in more detail.}
their prices, then productivity is realized, then monetary policy reacts and last the rest of the economy takes places where traded good firms set their prices and households make their decision.

The important feature in this setup is that a discretionary monetary authority has an incentive to use surprise-inflation to inflate away the socially inefficient markups of firms. Firms anticipate the attempt of the central bank to inflate and raise their prices for non-traded goods before. In equilibrium, the economy ends up with higher prices. A lack of commitment by the central bank results in an inflationary bias for the economy. In contrast, a central bank that commits to policies realizes that it cannot inflate away the markups. Hence it promises ex ante to focus on productivity shocks only when using monetary policy and successfully avoids the inflationary bias.

Countries can be identified by the history of country-specific shocks \( v^t = (v_0, v_1, \ldots, v_T) \) and are therefore symmetric with respect to their parameters, technology and preferences. We first consider how the economy works for one single “home” country and then consider country blocks and unions in Section 2.3.

### 2.1.1 Production

Firms are owned by households. Production of traded goods is given by

\[
Y_T(s^t) = L_T(s^t).
\]

Production is linear in the labor input \( L_T(s^t) \). Traded good firms maximize their profits \( P_T(s^t) L_T(s^t) - W(s^t) L_T(s^t) \). Optimally firms set the price of traded goods \( P_T(s^t) \) equal to the wage \( W(s^t) \). \( W(s^t) \) can therefore be replaced by \( P_T(s^t) \).

Production of non-traded goods is subject to two frictions, namely monopolistic markets and rigid prices. This gives rise to markups that increase prices of non-traded goods. A microfoundation for markups can be given by closely following the setup of Smets and Wouters (2007) which is also described in the Appendix of Chari et al. (2020a). The non-traded good is produced by a competitive final producer who uses differentiated inputs \( y_N(j, s^t) \) from input firms of mass \( j \in [0, 1] \) to produce the final good \( Y_N(s^t) \):

\[
Y_N(s^t) = \left[ \int y_N(j, s^t)^{\theta(s_{1[t]})} \, dj \right]^{1/\theta(s_{1[t]})}, \quad \theta(s_{1[t]}) \in (0, 1).
\]
where $\theta(s_{1t})$ is the time-varying substitution parameter between the inputs. $\theta(s_{1t}) \in (0, 1)$ implies that demand for inputs is elastic. If $\theta(s_{1t})$ is very close to 1 goods are almost perfect substitutes and firms are not able to use any monopolistic power. The closer $\theta(s_{1t})$ is to 0, the more monopolistic power a firm has. The final good firm maximizes

$$PN(s_{t-1},s_{1t})YN(s^t) - \int PN(j,s_{t-1},s_{1t}) yN(j,s^t) dj.$$ 

Demand for intermediate goods is therefore

$$yN(j,s^t) = \left( \frac{PN(s_{t-1},s_{1t})}{PN(j,s_{t-1},s_{1t})} \right)^{\frac{1}{\theta(s_{1t})}} YN(s^t).$$

Intermediate goods are produced by monopolistic firms who use a linear production function:

$$yN(j,s^t) = A(s_{2t})LN(j,s^t).$$

Intermediate good firms choose their prices $P = P(j,s_{t-1},s_{1t})$ to maximize their expected profits:

$$\max_P \sum_{s_{2t}} Q(s^t) \left[ P - \frac{W(s^t)}{A(s_{2t})} \left( \frac{PN(s_{t-1},s_{1t})}{PN(j,s_{t-1},s_{1t})} \right)^{\frac{1}{\theta(s_{1t})}} YN(s^t) \right]$$

where $Q(s^t)$ is the discount factor, the price of a state-contingent claim to local currency units at $s^t$ in units of local currency in $s_{t-1}$. Optimally, intermediate good producer $j$ sets the price on non-traded goods as a time-varying markup over a weighted average of marginal costs:

$$PN(j,s_{t-1},s_{1t}) = \frac{1}{\theta(s_{1t})} \sum_{s_{2t}} Q(s^t) YN(s^t) \left( \frac{W(s^t)}{A(s_{2t})} \right)^{\frac{1}{\theta(s_{1t})}} YN(s^t).$$

where $\frac{1}{\theta(s_{1t})}$ is the markup that increases prices. Note that the price equation is not a function of $j$ such that the price is the same for all intermediate firms. Plugging in $W(s^t) = PT(s^t)$ gives the pricing equation

$$PN(s_{t-1},s_{1t}) = \frac{1}{\theta(s_{1t})} \sum_{s_{2t}} Q(s^t) YN(s^t) \left( \frac{W(s^t)}{A(s_{2t})} \right)^{\frac{1}{\theta(s_{1t})}} YN(s^t).$$

(1)

This implies that all intermediate firms hire the same amount of labor and their production function is then simply given by

$$YN(s^t) = A(s_{2t})LN(s^t).$$

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\(2\)The elasticity of substitution between the inputs is \(\frac{1}{\theta(s_{1t})}\).
2.1.2 Households

Households derive utility from consumption of traded goods $C_T(s')$ and from consumption of non-traded goods $C_N(s')$. In addition, they experience disutility from labor $L(s')$: $\sum_{t=0}^{\infty} \sum_{s'} \beta^t h_t(s') U(C_T(s'), C_N(s'), L(s'))$.

As in Chari et al. (2020a), we specialize preferences as

$$U(C_T, C_N, L) = \alpha \log C_T + (1 - \alpha) \log C_N - \psi L.$$ 

This specification entails several simplifying assumptions, first it assumes that the elasticity of substitution between traded and non-traded goods is 1. Second, log-utility in consumption means that the inter-temporal elasticity of substitution is 1 as well. Those assumptions imply that households do not have an incentive to borrow or save across countries, as the willingness to substitute goods across time is exactly offset by the willingness to substitute traded goods to non-traded goods. $\alpha$ reflects the weight of traded goods in the overall consumption basket, large values imply that the countries in the economy have a very high degree of trade openness. Finally, the preferences are quasi-linear in labor, which simplifies aggregation results\(^3\).

The budget constraint of households is given by

$$P_T(s') C_T(s') + P_N(s'^{-1}, s_{1t}) C_N(s') + M_H(s') + B(s') \leq P_T(s') L(s') + M_H(s'^{-1}) + R(s') B(s'^{-1}) + T(s') + \Pi(s')$$

where $T(s')$ are nominal transfers. $\Pi(s') = P_N(s'^{-1}, s_{1t}) Y_N(s') - P_T(s') L_N(s')$ are profits from the traded-goods sectors. As households own the firms in their corresponding country, these profits go to the households. Firms themselves are not traded on international markets. $R(s')$ is the interest rate paid on the non-contingent one-period nominal bond in the domestic currency and $B(s')$ are the nominal government bonds. Compared to Chari et al. (2020a), we replaced the price that is paid to buy new bonds with interest rates that are paid on existing bonds. We show in the Appendix B.1 that the price of bonds in Chari et al. (2020a) is simply the inverse of interest rates used here. The model abstracts from international capital markets, as households do not have an incentive to borrow or lend across countries, given the considered preferences.

There is also a cash-in-advance constraint for consumers, that requires domestic money brought from period $t - 1$ to be used to purchase traded goods:

$$P_T(s') C_T(s') \leq M_H(s'^{-1})$$

\(^3\)Quasi-linear utility eliminates any wealth effects in the demand for money, which makes all agents choose the same amount of money. See Ricardo and Wright (2005)
Under flexible exchange rates, consumers use their local currency $M_H(s^{t-1})$ to pay for these goods. The superscript $H$ denotes the individual holding of money. Domestic money is only held by domestic households. Even though money is dominated by bonds as they pay interest on the existing stock, households need money to buy traded-goods. The assumption of cash-in-advance makes surprise inflation costly, as they can only use cash from the last period. In addition, the assumption that only traded goods are affected by this is for simplicity. This assumption can also be interpreted as a trade friction that requires to commit a certain amount of cash beforehand when internationally traded goods are bought from a foreign country. Note that current money injection that increase the nominal price of traded goods cannot be used for the cash in advance constraint. In a currency union they use the common currency to pay for the traded goods.

The first order conditions for the households imply

\[
\frac{U_N(s^t)}{P_N(s^{t-1},s_{1t})} = -\frac{U_L(s^t)}{W(s^t)} + \frac{U_T(s^t)}{P_T(s^t)},
\]

\[
\frac{U_T(s^t)}{P_T(s^t)} = -\frac{U_L(s^t)}{W(s^t)} + \phi(s^t),
\]

\[
\frac{U_L(s^t)}{W(s^t)} = \beta \sum_{s^{t+1}} h(s^{t+1} | s^t) \frac{U_T(s^{t+1})}{P_T(s^{t+1})},
\]

\[
1 = \beta \sum_{s^{t+1}} h(s^{t+1} | s^t) R(s^{t+1}) \frac{U_N(s^{t+1})}{P_N(s^t, s_{1t+1})} \frac{P_N(s^{t-1},s_{1t})}{U_N(s^t)}.
\]

where $\phi(s^t)$ is the normalized multiplier of the cash-in-advance constraint. The Euler equation can be obtained by combining the home bonds first order condition with the consumption first order condition. It governs the household’s intertemporal decision:

\[
\frac{1}{C_N(s^t)} = \beta E_t \left[ \frac{1}{C_N(s^{t+1})} \frac{P_N(s^t)}{P_N(s^{t+1})} R(s^{t+1}) \right].
\]  

(3)

The nominal stochastic discount factor is defined as

\[
Q(s^{t+1}) = \beta h(s^{t+1} | s^t) U_N(s^{t+1}) P_N(s^{t-1},s_{1t}) / \left( P_N(s^t, s_{1t+1}) U_N(s^t) \right).
\]

This discount factor is also used by firms to discount their profits.
2.1.3 Government

The government budget constraint for each country under flexible exchange rates is given by

$$B(s^t) = R(s^t)B(s^{t-1}) + T(s^t) - (M(s^t) - M(s^{t-1})),$$

where $M(s^t)$ denotes the money supply in the economy. The last term is seignorage income from the growth in money supply. In a currency union, union-wide seignorage is equally split across countries according to their size. The initial money supply for each consumer in each country is set to $M_{-1}$ and the initial bond holding $B_{-1}$ are zero. The central bank specifies nominal interest rates, the quantity of debt and taxes for each country, satisfying the budget constraint. Note that there are no externalities for the central banks. This ensures that monetary policy does not have any incentive to set monetary policy in a non-cooperative way and to use its monopoly on its currency to manipulate the terms of trade.

2.2 Market Clearing and Equilibrium

Labor markets clear, which means that the demand for non-traded goods labor and traded goods labor equals overall labor supply

$$L_N(s^t) + L_T(s^t) = L(s^t).$$

Good markets clear for traded and non-traded goods.

$$C_T(s^t) = Y_T(s^t), \quad C_N(s^t) = A(s^t)Y_N(s^t).$$

GDP in this model is defined as the sum of consumption of traded and non-traded goods. Money demand from households $M_H(s^t)$ is met by money supply of the central bank

$$M_H(s^t) = M(s^t).$$

An equilibrium under flexible exchange rates is defined as an allocation in which 1) consumers behave optimally, 2) firms behave optimally, 3) the government’s budget constraint holds and 4) markets clear.

As the law of one price holds in this model, the multilateral exchange rate can be defined as the price of traded goods in the considered country relative to the average price of traded goods.
in the rest of the world:

\[ e(s^t) = \frac{P_T(s^t)}{\sum_{v^t} P_T(z^t, v^t) g^t(v^t)}, \]

where \( g^t(v^t) = g(v_0)...g(v_t) \) is simply the average over all countries. With a sufficiently large rest of the world, only country-specific shocks of the considered country can change the exchange rate, as the common shocks are the same and the average of the price of traded goods in the rest of the world is independent of shocks to small countries in the rest of the world.

In a monetary union money supply is chosen by the union-wide central bank. The nominal exchange rate is fixed for all states: \( e(s^t) = 1 \) \( \forall s^t \) and consequently, the price of traded goods is the same everywhere. This means that only aggregate shocks can change the price of traded goods. Formally, if the state of the world in one country is \( s^t = z^t, v^t \) and \( \tilde{s}^t = z^t, \tilde{v}^t \) in the other country, then prices of traded goods are still the same

\[ P_T(s^t) = P_T(\tilde{s}^t). \]

An equilibrium in a monetary union is defined in the same way as with flexible exchange rates, the only difference being that the exchange rate is set to 1 for all states and that total money holding in a union adds up to the overall money supply

\[ \sum_{v^t} M_H(z^t, v^t) g^t(v^t) = \bar{M}(z^t). \]

In this model, shocks to markups lead to distortions in the economy that vary over time. This can be seen when combining the first order conditions of households with the first order condition of firms. Suppose productivity is constant, then the marginal rate of substitution (MRS) between labor and non-traded goods equals the marginal rate of transformation (MRT) of labor times the inverse markup

\[ -\frac{U_L}{U_N} = A \theta(s_t) < A. \]

This means that the markup drives a wedge \( 1 - \theta(s^t) \) between the MRS and the MRT. The larger the markup \( 1/\theta(s^t) \), the greater the distortions resulting from imperfect competition. The next section explains how monetary policy deals with that issue and how a lack of commitment can lead to an inflationary bias in that environment.
2.3 Monetary Regimes

This subsection discusses the equilibrium of real and nominal variables under different monetary regimes. We still follow Chari et al. (2020a) and consider three regimes: A floating regime with flexible exchange rates, a unilateral peg with a fixed exchange rate and a currency union. A country can conduct monetary policy under commitment and under discretion. We then extend the model and include a credibility parameter as in Schaumburg and Tambalotti (2007) that governs the probability of being in a discretionary regime. The interpretation is that a new governor gets selected with probability $\xi_t$ in every period. If a new governor is selected, she acts under discretion in the first period and commits to policy thereafter as long as she is in office. It is not possible to restrain the successor. Formally, there is a sequence of Bernoulli signals $\eta_t$, with probability $\xi$, $\eta_t$ is one and a new governor is chosen, otherwise $\eta_t$ is zero and the old governor stays in place. We assume that this signal is known before productivity has realized. This implies that firms know if monetary policy acts under commitment or under discretion in a certain period. The timing of the model then looks like this:

The central bank ends up with a policy rule that is either discretionary or commitment based. Firms set their prices accordingly to each regime. As the signal is i.i.d. and there are no other state variables so far, the solution to the model under each regime separately is not affected. The average value of variables over a long time horizon is changed however. Average inflation for example is then the weighted average of inflation under discretion and under commitment. The weights correspond to the credibility parameter $\xi_t$ that determines the probability of acting under discretion. This probability is be time-varying ($\xi_t$). We will estimate this time-varying probability in Section 3. If a country in this setup decides to peg its currency to a stable anchor, the probability of being in a discretionary regime decreases to the level of the anchor country. Such modeling decision is motivated by the facts that countries with safer currencies enjoy persistently lower interest rates and a lower required return to capital (Hassan and Zhang, 2021) and thus have less incentive to act in discretion, and that pegging to a stable anchor currency present the additional advantage of coordinating expectations (Levy-Yeyati and Sturzenegger, 2010). In a currency union, the central bank is as credible as the most credible member state. Next, we describe how policy in each regime under discretion and commitment looks like. These results reproduce those in Chari et al. (2020a).
2.3.1 Flexible Exchange Rates: Monetary Policy under Commitment

The central bank conducts monetary policy under commitment. This means that the central bank maximizes ex ante lifetime utility of its representative household. It chooses an appropriate state-contingent path of prices subject to the consumer and firm first order conditions, the resource constraint, as well as the production function. The central bank sets its policy after productivity has realized. Importantly, the central bank realizes that firms will set their relative prices equal to expected productivity times the markup. In a world under discretion, in which the central bank would take $P_N(s^{t-1}, s_{1t})$ as given, it would try to inflate away the markup, to set $P_T(s^t)/P_N(s^{t-1}, s_{1t}) = A(s_{2t})$. Under commitment the central bank realizes that this attempt of surprise inflation will not work. Therefore, optimal policy does not respond to markup shocks. It only responds to productivity shocks. Intuitively, the monetary authority has to live with the distortions from markup shocks and attempts to accommodate productivity shocks. Therefore, the optimal policy of the central bank implies

$$\frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})} = \theta(s_{1t})A(s_{2t}).$$

The interpretation of that policy rule is straightforward: After productivity has realized the central bank makes sure that relative prices move in such a way that they replicate the outcome as if non-traded good prices were flexible. This way the central bank can eliminate any distortions coming from rigid prices. The central bank engineers a movement of the exchange rate in such a way that relative prices align. For example, if productivity of the non-traded goods sector is high today, $P_N$ should decrease as it is easier to produce that good. As prices of that good do not adjust, the central bank instead uses the exchange rate to let the currency depreciate so such $P_T$ rises, which means that the relative price for $P_N$ falls. The movement of the exchange rate aims to replicate the outcome of relative prices as if all prices were flexible. Note also, that optimal monetary policy would never cause consumers to lose consumption because they do not have enough cash. Therefore, the cash in advance constraint is never binding in a way that would lower the household’s consumption. That is the reason why the consumer first order condition with respect to $C_T$ has a multiplier from the cash in advance constraint equal to zero.

2.3.2 Flexible Exchange Rates: Monetary Policy under Discretion

Now consider how a non-credible central bank sets monetary policy. The important difference when a central bank acts under discretion is that it takes the price of non-traded goods as

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*For details, see the Appendix B.4 or Chari et al. (2020a).*
given, as firms have set their prices before the central bank acts. As a consequence, the central bank will try to use monetary policy to inflate away the inefficient monopolistic markups and implement an allocation, that equalizes household’s marginal rate of substitution with the marginal rate of transformation of the economy. That is $P_T(s^t)/P_N(s^{t-1}, s_{1t}) = A(s_{2t})$. In order to do that the central bank will go so far to make the cash in advance constraint binding. As long as this constraint is slack, the central bank can use more inflation to reduce the markups. Therefore, the central bank makes the cash in advance constraint binding and ultimately trades off the costs of markups with the costs of surprise inflation that lower the household’s purchasing power. For further details of the optimization problem, see Appendix B.4. The best response of the monetary authority is to set the price of traded goods as:

$$p_T(s_t) = p_N(s_{1t}) A(s_{2t}) \frac{1}{2(1-\alpha)} \left[ (1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha) \frac{1}{A(s_{2t}) p_N(s_{1t})}} \right]$$

where the first part on the right-hand side $p_N(s_{1t}) A(s_{2t})$ captures the willingness of the central bank to put the marginal rate of transformation equal to the marginal rate of substitution and $F(\cdot)$ captures the costs from surprise inflation. If $p_N$ increases by one, $p_T$ increases less than one-to-one. In the following we assume as in Chari et al. (2020a) that $\frac{1}{\theta(s)} < \frac{1-\alpha}{1-2\alpha}$ so that there is a point where marginal costs of surprise inflation equal their marginal benefits. This simply bounds markups from above, meaning that it is not possible that reducing markup distortions always exceed the costs of reducing trade goods consumption.

Another aspect that needs to be mentioned is, when productivity is stochastic and is sufficiently low compared to its average value, it can happen that the cash in advance constraint is not binding despite the central bank’s policy. That is if $Ap_N < C_T$ then $p_T = p_N A$. Taken this into account as well, it implies that the price of traded goods is described by $p_T(s_t) = \max\{p_N(s_{1t}) A(s_{2t}), p_N(s_{1t}) A(s_{2t}) F(\cdot)\}$.

For policy under discretion, it is also important to consider the firms. They take into account that the central bank will try to inflate away their markups. Optimally firms still set prices of traded goods as in (1). Remember that firms observe the markup shock and then set their price taking their expectation for future productivity into account. Overall, the price of traded goods in the equilibrium solves the fixed-point problem of equaling the optimal price firms would set and what the central bank wants to implement. So, in equilibrium, any attempt of the central bank to inflate away the markup is frustrated, as firms anticipate the central bank’s move and set their prices accordingly. The only thing the central bank achieves is an inflationary bias with higher volatility of prices and consumption.
2.3.3 Unilateral Peg to an Anchor

Consider now the case in which one country (the client country) pegs its currency to another country (the anchor). The anchor is assumed to conduct monetary policy under commitment or discretion, as in Section 2.3.1 and 2.3.2. The client country then ensures that the exchange rate to the anchor country stays constant at all points in time. This implies that monetary policy of the client loses its independence and follows the anchor. The main difference to this regime and a currency union is that the client country has no influence how the anchor conducts monetary policy. In a currency union the union-wide central bank considers all its member states. The peg implies that the price of traded goods is the same for both countries. Firms of the client country realize that monetary policy will be as in the anchor country. After markup shocks have realized in the anchor country, they form expectations about productivity and how the central bank of the anchor chooses the price of traded goods. In general, distortions coming from productivity fluctuations will be completely offset in the anchor country, while they will be present in the client country. These distortions are reflected in a volatile movement of employment. There can be an inflationary bias in both countries if the anchor acts under discretion a certain period.

2.3.4 Currency Union: Monetary Policy under Commitment

In a monetary union, the exchange rate is fixed and set to $e(s^t) = 1$ for all states. This implies that $P_T$ cannot vary across countries and is only a function of aggregate union-wide shocks. This gives rise to the “Union constraint”

$$\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\bar{s}^t)}{U_N(\bar{s}^t)}$$

The union consists out of many blocks, each block $i$ having a mass of countries $n^i$. The relative weight of block $i$ is $\lambda^i = \frac{n^i}{\sum n^i}$. Countries are all the same across blocks, except for the shock process of their markup. The central bank acts under commitment and chooses the union-wide price of traded goods and the prices of non-traded goods to maximize an equally weighted average of all countries of the world. Optimally, the cash in advance constraint does not bind to avoid losses in consumption as in the case under commitment before. The central bank sets prices such that it stabilizes the average of the whole union:

$$\frac{p_T(s^t)}{p_N(s^{t-1}, s_{1t})} = \theta^i(s_{1t}) \left( \sum_i \lambda^i \sum_{v^{2t}} g(v^{2t}) \frac{1}{A^i(z^{2t}, v^{2t})} \right)^{-1},$$
where \( g(v) \) sums up within the blocks of a currency union and \( \lambda \) sums up the blocks of the union. As the exchange rate is fixed, prices of traded goods are the same for all countries and the only thing the union-wide central bank can do is to set relative prices equal to the markup times the average productivity of the union. The price of non-traded goods fluctuates together with markups and the central bank under commitment does not react to that.

### 2.3.5 Currency Union: Monetary Policy under Discretion

The same union constraint applies for policy under discretion in a currency union. The central bank acts under discretion and chooses the union-wide price of traded goods to maximize an equally weighted average of all countries of the world. The union-wide central bank chooses a traded good price for the union taking the non-traded good prices as given. The policy of the central bank implies to set the price of traded goods such that:

\[
p_T(z, \{ p_N^i(z_1, v_1) \}) = \frac{(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4 \sum_i \lambda^i \sum_v g(v) \frac{(1 - \alpha) A(z_2, v_2)}{A(z_2, v_2) p_N^i(z_1, v_1)}}}{\sum_{i=N,S} \lambda^i \sum_v g(v) \frac{2(1 - \alpha) A(z_2, v_2)}{A(z_2, v_2) p_N^i(z_1, v_1)}}.
\]

where \( g(v) \) gives the average state of all countries within a block, given the aggregate state. Compared to the policy rule under discretion with an independent national central bank single country-specific shocks are replaced by the average shock realization of the union.

As before, firms anticipate the policy of the central bank and take this into account when setting their prices. In a currency union however, they realize that the central bank will only react to the average temptation shock, not the country-specific one. The result is still more inflation. The next section discusses how the policy under discretion in a currency union can still yield some benefits compared to discretion of a single country.

### 2.4 Overview

This section summarizes key real and nominal variables given the policy rules under different monetary regimes. We still closely follow Chari et al. (2020a), the model results under each regime are the same as in their Appendix 7 and 10 I (Chari et al., 2020b) with blocks of countries that form a currency union.

The following table summarizes the regimes and how we match those regimes to the classification in Section 5.1.

| Table 1: Monetary regimes Model and Data. |
For simplicity, we focus on a model solution with productivity such that the cash in advance constraint is exactly binding in discretion. First, turn to the nominal variables of the model. Table 2 shows average nominal interest rates under different regimes, while Table 3 shows average inflation of non-traded goods.\(^5\).

Table 2: Average nominal interest rates under different monetary regimes.

<table>
<thead>
<tr>
<th>Model Regime Classification</th>
<th>Probability</th>
<th>Empirical Regime Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float &amp; Commitment</td>
<td>(1 - \xi_t)</td>
<td>Peg</td>
</tr>
<tr>
<td>Float &amp; Discretion</td>
<td>(\xi_t)</td>
<td>Peg</td>
</tr>
<tr>
<td>Peg &amp; Commitment</td>
<td>(1 - \xi_t^{\text{Anch}})</td>
<td>Peg</td>
</tr>
<tr>
<td>Peg &amp; Discretion</td>
<td>(\xi_t^{\text{Anch}})</td>
<td>Peg</td>
</tr>
<tr>
<td>Union &amp; Commitment</td>
<td>(1 - \min{\xi_t^i})</td>
<td>Union</td>
</tr>
<tr>
<td>Union &amp; Discretion</td>
<td>(\min{\xi_t^i})</td>
<td>Union</td>
</tr>
</tbody>
</table>

Table 3: Average inflation rate under different regimes for state \(s\).

<table>
<thead>
<tr>
<th>Regime</th>
<th>(\pi_N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float</td>
<td>((1 - \xi_t^{\text{Anch}}) \frac{\theta(s)}{\theta(s')} \beta + \xi_t^{\text{Anch}} \frac{\theta(s)}{\theta(s')} \beta + \xi_t \frac{\theta(s)}{\theta(s')} \beta \frac{\alpha}{\alpha - (1 - \alpha)(1 - \theta(s'))} \Theta(s'))</td>
</tr>
<tr>
<td>Peg</td>
<td>((1 - \xi_t^{\text{Anch}}) \frac{\theta(s)}{\theta(s')} \beta + \xi_t^{\text{Anch}} \frac{\theta(s)}{\theta(s')} \beta \frac{\alpha}{\alpha - (1 - \alpha)(1 - \theta(\text{Anch}(s)))} \Theta^{\text{Anch}}(s'))</td>
</tr>
<tr>
<td>Union</td>
<td>((1 - \min{\xi_t^i}) \frac{\theta(s)}{\theta(s')} \beta + \min{\xi_t^i} \frac{\theta(s)}{\theta(s')} \beta \frac{\alpha}{\alpha - (1 - \alpha)(1 - \sum s_i \lambda^i \theta(s'))} \Theta^U(s'))</td>
</tr>
</tbody>
</table>

Notes: Average inflation of non-traded goods (\(\pi_N\)) under all regimes. Average inflation is the weighted average under discretion with probability \(\xi_t\) and under commitment with probability \((1 - \xi_t)\). In a currency union there are blocks of countries each with a mass \(\lambda^i\).

\[\Theta(s') = \frac{\alpha}{\alpha - (1 - \alpha)(1 - \theta(s'))}\] and \[\Theta^U(s') = \frac{\alpha}{\alpha - (1 - \alpha)(1 - \sum s_i \lambda^i \theta(s'))}\]. Under commitment, the gross nominal interest rate is one, which means that nominal interest rates are zero. The central bank follows the Friedman (1969) rule implying a negative money growth rate. The

\(^5\)For a derivation see the Appendix B.4 and Table B.2
intuition why zero interest rates are optimal under commitment is the following. For households, nominal bonds dominate money holding as long as they pay an interest on its stock. Money does not pay any returns for its holder. Nevertheless, households need to hold money to buy traded goods. Therefore, the central bank optimally implements zero interest rates to make the necessary money holding as good as the bond holding. In addition, deflation ensures that the cash in advance constraint is never binding for households.

In contrast inflation, interest rates and money growth rates are larger in discretionary regimes. As discussed before, the central bank has an incentive to use surprise inflation to inflate away markups. Ultimately, the central bank trades off costs of inflation in form of a binding cash in advance constraint with reduced markups. Firms anticipate this attempt and simply raise their prices. In equilibrium, the economy ends up with higher inflation. The size of the inflationary bias depends on \( \frac{\alpha}{\alpha - (1 - \alpha)(1 - \theta(s))} \). Values of that term close to one imply no inflationary bias. This means that larger markups (small \( \theta(s_U) \)) correspond to a larger inflationary bias. The larger trade-openness (large \( \alpha \)) the lower is the inflationary bias. As internationally traded goods are more important to households, the central bank is careful not to induce too much inflation that lowers consumption of internationally traded goods. The central bank achieves higher inflation by inducing a positive growth rate for money supply. The Euler equation then dictates that nominal interest rates have to be higher as well. As before, the average level of inflation and interest rates is a weighted sum of the values under different regimes with the credibility parameter \( \xi_t \) determining the likelihood. The first theorem summarizes the implication for the level of inflation and interest rates when a country pegs to another country.

**Theorem 1** If a country peg its currency to a more credible anchor country, its inflation and interest rates fall permanently. If a group of countries form a currency union, the level of inflation and interest rates falls permanently for all countries that are less credible than the most credible member states.

Proof: See Appendix B.5. Next, consider the role of \( \Theta(s) \) that impacts inflation under discretion: This term adds more volatility in the inflation process. If the markup rises in the future, this also increases inflation of this good by a larger amount. If markups are lower than usual, then inflation decreases more than without this term. It is simply an amplifier. Together with the higher money growth rate, inflation rates are higher on average and more volatile in a discretionary float. A currency union can ensure that \( \Theta \) is more stable over time when countries with the same markup shock process form a union. Country-specific markup shocks vary more than the average of all markup shocks. Therefore, a currency union is able to reduce the
volatility of inflation not only because the frequency of discretionary regimes is reduced, but also because in times of discretion monetary policy for the whole union is less erratic. For the anchor country another effect is important too: As the growth rate of its markup is less correlated with $\Theta^U$ than with $\Theta^{Anch}$, inflation volatility goes down for it even more. This leads to the second theorem that we can test with the data:

**Theorem 2** If a group of countries form a currency union, the volatility of inflation and interest rates go down permanently when markup shocks are not perfectly correlated. This is also true for the former anchor country.

The proof can be found in the Appendix B.5. Furthermore, average money growth rate in a currency union must be lower as well, as $\alpha - [1-\alpha][1-\theta(s)]$ is a convex function in $\theta$. The average value of that term with country-specific shocks is larger as the average value of that term with average union-wide shocks. This implies that -absent stochastic productivity- forming a currency union for countries with the same stochastic process yields benefits, as inflation is lower on average.

Last, consider how output compares across the three regimes$^6$

<table>
<thead>
<tr>
<th>Regime</th>
<th>$Y_T$</th>
<th>$Y_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float</td>
<td>$(1 - \xi_t) \frac{\alpha}{\psi} + \xi_t \left(\frac{\alpha}{\psi} - \frac{1-\alpha}{\psi} \left(1 - \theta(s)\right)\right)$</td>
<td>$\frac{1-\alpha}{\psi} \theta(s) A(s)$</td>
</tr>
<tr>
<td>Peg</td>
<td>$(1 - \xi_t^{Anch}) \frac{\alpha}{\psi} + \xi_t^{Anch} \left(\frac{\alpha}{\psi} - \frac{1-\alpha}{\psi} \left(1 - \theta(s)\right)\right)$</td>
<td>$\frac{1-\alpha}{\psi} \theta(s) E(v(1/A(s))^{-1})$</td>
</tr>
<tr>
<td>Union</td>
<td>$\left(1 - \min{\xi_t^{Anch} } \right) + \min{\xi_t} \left(\frac{\alpha}{\psi} - \frac{1-\alpha}{\psi} \left(1 - \sum \lambda_i E(v(\theta_i(s))\right)\right)$</td>
<td>$\frac{1-\alpha}{\psi} \theta(s) \left(\sum \lambda_i E(v(1/A_i(s))^{-1})\right)$</td>
</tr>
</tbody>
</table>

Notes: Average output of traded goods ($Y_T$) and non-traded goods ($Y_N$) under all regimes. Average output of traded goods is the weighted average under discretion with probability $\xi_t$ and under commitment with probability $(1 - \xi_t)$. In a currency union there are blocks of countries each with a mass $\lambda$.

In general, output of traded goods is larger the larger the trade openness $\alpha$. Large values of disutility from work $\psi > 0$ lower output. In general, the average output of traded goods is a weighted average of output under a discretionary regime and commitment. With probability $1 - \xi_t$ there is a commitment-based regime, while with probability $\xi_t$ there is discretion. Under discretion $C_T$ is lower than with commitment, as the central bank follows an inflationary policy. With high inflation, the household’s cash in advance constraint is binding such that traded good consumption is lower, implying lower output. Larger markups increase the inflationary bias and hence decrease the amount of traded goods output under discretion. That is, if $\theta \in (0, 1)$ is relatively small. If a country follows a unilateral peg, the probability of being

$^6$All six regimes separately are reported in the Appendix, Table B.1.
in a regime with high inflation is $\xi_t^{Anch}$, the probability that the anchor country’s central bank acts under discretion in a period. If the anchor country is more credible, average output will be higher. If a currency union is formed the union-wide central bank becomes as credible as the most credible member state. The following proposition summarizes a testable implication for output under different regimes:

**Theorem 3** If a country pegs its currency to a more credible country, output rises. If a group of countries form a currency union, output of all countries where inflation goes down rises.

Output of non-traded goods is a function of actual productivity and markups, as long as the exchange rate can float. If the client country pegs its currency, output on non-traded goods becomes a function of expected productivity, as the anchor central bank does not react to productivity of the client country. As soon as there is a currency union non-traded good output is a function of the average productivity of the union: With a common currency the central bank can only ensure that relative prices align for the average of the union, not for each individual country. If productivity is stochastic, there will be welfare losses in a peg: Prices for very productive countries are too high compared to a flex price world, while prices of low productivity countries are too low. This is inefficient as this implies employment gaps that lower welfare.

The next chapter calibrates the shock process in more detail.

### 3 Calibration

In this section, we describe the calibration strategy for the economic model and report the calibrated parameter values and the empirical targets matched by the model.

#### 3.1 Calibration Strategy

The model seeks to highlight the effects of fixing the exchange rate via a unilateral peg or forming a currency union. Towards that aim, we focus on Germany and Italy between 1960 to today. During this time horizon, both countries go through various different exchange rate regimes. That sample includes the time after the breakdown of Bretton Woods in which the exchange rate of Italy moved by a great margin. In 1985 Italy decided to peg its currency to the German Mark. In the end of the 90s, both countries formed the European currency union together with other European countries. The reason why we focus on Germany and Italy is that they are the largest countries of their respective block: Germany being part of the core (or the northern) block in the currency union, with relatively low and stable inflation
rates before. And Italy as the largest country of the periphery (or the southern block) that experienced large increases in inflation during the mid 70s and 80s. One period in the model taken to be a year. The calibration proceeds in two steps. First, we calibrate parameters based on long-run moments in the data and the outside literature. Thereafter, taking these as given, we calibrate the process for markup shocks and the credibility parameter to match key stylized facts on the property of inflation for Germany and Italy.

The model is kept relatively simple, therefore only a couple of parameters need to be calibrated. This table summarizes the directly calibrated parameters:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>Time discount rate</td>
<td>Real rate of 2% p.a.</td>
</tr>
<tr>
<td>$\psi$</td>
<td>8/3</td>
<td>Disutility of work</td>
<td>1/3 of time spent working</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
<td>Share traded consumption goods</td>
<td>Trade openness Germany 2015</td>
</tr>
</tbody>
</table>

The time discount factor is chosen to replicate a real interest rate of around 2% per year, in line with estimates for European countries by Brand et al. (2018). Next, we choose $\alpha$ – a measure of trade openness– to be 35 % in line with imports over GDP for Germany in 2015. We also consider the impact of smaller values of trade-openness in Figure B.1. The trade elasticity and intertemporal elasticity is already chosen to be 1 in the specification of preferences.

Next, we turn to the heart of the calibration, that aims to match cyclical inflation movements in Europe with the evolution of markups and the credibility parameter in the model. We will consider a model under a floating exchange regime and compute its moment. This way, we can assess whether the estimated evolution of credibility is consistent with credibility in the model under different regimes, e.g. whether Italy’s credibility indeed approached the German level, when it decided to peg its currency.

Calibrating he credibility parameter $\xi_t$ is crucial, as it determines how often a country is in a discretionary regime. This impacts average inflation over a considered time horizon. This credibility parameter is country-specific and time-varying. Next to this, the markups process $1/\theta(s)$ is important too. It determines how large and volatile the inflationary bias is on for those countries. The range of estimates of markups varies widely, see for example De Loecker and Warzynski (2012), Christopoulou and Vermeulen (2012), Kuester (2010) or Midrigan (2011). In most applications, as for instant in Gomes et al. (2012), markups vary between 15% and 50%. The higher the markup, the higher inflation under discretion. For more open economies -larger
\(\alpha\)-inflation is lower. In our model, relatively low markups already lead to very high inflation values under discretion, see Figure B.1. Therefore, to avoid unreasonably high inflation rates, we aim for a macro-markup of 7% for both countries which is substantially lower what the literature usually chooses. The goal is to match the behavior of inflation using the simulated method of moments: The model generates certain moments of inflation given a process for \(\theta(s)\) and \(\eta(s)\), like the mean and volatility of inflation in a float. The model predicts that countries in a float have potentially different inflation rates, depending on their shock process and credibility. We then assume that the country-specific component of \(\theta\) is beta distributed between 0 and 1 with parameters \(\bar{\beta}\) and \(\bar{\beta}\). We estimate these two parameters at each point in time such that the shape of the distribution for markup shocks fits the data well. The global component is muted for this exercise, the process is still assumed to be i.i.d. The same applies for \(\eta_t\), we estimate the probability \(\xi_t\) of acting under discretion in a given period. I.i.d is an important assumption as there is no persistent component in the process that we estimate. Large shocks today do not have an impact on future shocks, if a country is discretionary today is has the same chance to be discretionary tomorrow. Even though this assumption limits the behavior of markups and regimes, it keeps the model simple and tractable. For now, we also impose zero correlation of these shocks between countries. We aim to include correlation between countries in the future, as the correlation of markup shocks is a crucial component for the desirability of currency unions. Our method will then choose the two parameters of the beta distribution and the credibility parameter for each country separately for each year. A country with very volatile inflation will require a flatter distribution of \(\theta\) and to have lower credibility. Low average inflation values would correspond to relatively low markups, that is a distribution of theta that centers around a value close to 1, together with a high degree of credibility. We also impose that certain model assumptions still hold, such as \(\theta > 1 - \alpha/(1 - 2\alpha)\). For further details how SMM works, see Appendix B.8.1

### 3.2 Calibration Results

Next, the table summarizes the mean estimation, the moments of the data and the moments of the model under a discretionary float for both countries.

<p>| Table 6: SMM Calibration |</p>
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Italy</th>
<th>Germany</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>76.55%</td>
<td>53.48%</td>
<td>mean credibility: Prob. of discretion</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
<td>37</td>
<td>37.14</td>
<td>Parameter 1 beta distribution</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
<td>791.24</td>
<td>818.76</td>
<td>Parameter 2 beta distribution</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu(\pi)$ data</td>
<td>5.62%</td>
<td>2.49%</td>
<td>Av. inflation 1950-end</td>
</tr>
<tr>
<td>$\mu(\pi)$ model</td>
<td>5.24%</td>
<td>1.38%</td>
<td>Av. inflation in the model</td>
</tr>
<tr>
<td>$std(\pi)$ data</td>
<td>0.019</td>
<td>0.01</td>
<td>Standard dev. inflation 1950-end</td>
</tr>
<tr>
<td>$std(\pi)$ model</td>
<td>0.029</td>
<td>0.02</td>
<td>Standard dev. inflation in the model</td>
</tr>
<tr>
<td>$\mu(\theta^{-1})$ data</td>
<td>7%</td>
<td>7%</td>
<td>Target markup</td>
</tr>
<tr>
<td>$\mu(\theta^{-1})$ model</td>
<td>4.9%</td>
<td>4.4%</td>
<td>Average markup in the model</td>
</tr>
</tbody>
</table>

The average of the estimated credibility parameter indicates that Italy is under discretion more often than Germany, Figure B.2 shows how calibrated credibility evolves over time for both countries and how it reacts under different regimes. Mean markups for both countries are between 4% and 5% on average and vary around this value. These lower markups coincide with a slightly too low inflation rate for both countries, that is mainly driven by the low inflation rates that the model predicts in a currency union\(^7\). On the other hand inflation volatility is too large in the model. This might reflect the lack of persistence in the shock process and in prices. This implies that switches between commitment- and discretionary policies cause a lot of variation in inflation.

4 Quantitative results

This section uses the calibration of the model to compute the model-based moments under all different regimes. In addition to that, mean inflation over time is computed, given the estimated parameters over time. We first report the moments of inflation in the model under the three regimes for Italy and Germany.

\(^7\)We assume that the size of the two countries in the currency union $\lambda^i$ is the same, such that the central bank puts an equal weight on both countries.
Table 7: Inflation under all regimes, model and data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
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<tr>
<td></td>
<td>mean</td>
<td>std. dev.</td>
<td>mean</td>
</tr>
<tr>
<td>Italy</td>
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<tr>
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<td>15.2%</td>
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<td>4.8%</td>
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<td>$\xi$ (SMM)</td>
<td>98.36%</td>
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<tr>
<td>Germany</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi$ data</td>
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<td>0.02</td>
<td>2.1%</td>
</tr>
<tr>
<td>$\pi$ model</td>
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<td>0.03</td>
<td>1.3%</td>
</tr>
<tr>
<td>$\xi$ (SMM)</td>
<td>87.87%</td>
<td></td>
<td>48.19%</td>
</tr>
</tbody>
</table>


For Italy, inflation after the collapse of Bretton woods was very high, both in the model and in the data. This coincided with a very large probability of acting under discretion. The central bank was not credible at all and there is a huge inflationary bias as a consequence. After Italy pegs its currency to Germany, its central bank becomes more credible, in fact nearly as credible as the German central bank was after the collapse of Bretton woods. Its inflation rates are also similar on average to the rates of Germany during the time of the float. For Germany in contrast, the time after Italy pegged its currency is characterized by even lower inflation rates, which the model achieves by assigning Germany a substantially more credible central bank for that time period. The creation of the currency union then leads to a substantial reduction in inflation and volatility again. Interestingly, even though both countries are subject to the same monetary policy in the data, the model assigns lower credibility to Italy, as its inflation rate is on average larger and still more volatile than Germany’s which is particularly true during the financial crisis 2009. The estimated model suggests that Italy managed to increase its credibility substantially over time. This coincided with moving towards a more fixed exchange rate regime with Germany. The same is however true for Germany, the original anchor. Its monetary authority got more credible as well over time.

Next, consider how the model replicates the evolution of inflation between 1950 and 2017, given the time varying parameters of credibility for Germany and Italy. We also plot the evolution of traded goods consumption.
Inflation is well tracked, until both countries prepare to enter the European currency union in the late 90s. Empirically, the period from the currency union onward is characterized by low levels of inflation, together with very low values of inflation volatility. The model matches these moments best, if it assigns nearly full credibility to the central bank. Full credibility in the model implies the Friedman rule for both countries. This means zero interest rates and negative inflation in the steady state. There are no costs of deflation in the model, which is why this is the outcome under commitment. The model then undershoots the empirical level target for inflation, matches however its volatility.

Traded good consumption is negatively correlated with inflation. The reason for that is, that inflation is costly and reduces traded goods consumption. Whenever a country is in a discretionary regime with high inflation rates, $C_T$ is lower. This is why we observe temporarily lower values of $C_T$ after the collapse of Bretton Woods in the early 70s, which subsequently rises again when lower inflation rates coincide together with more fixed exchange rate regimes.
Changes in the exchange rate regime have an impact on credibility in the model. When Italy pegs its currency to Germany, its monetary policy should become more credible and in fact as credible as the one of Germany. The estimated path of $\zeta_t$ suggests that this is true, see Figure B.2 and Table 7. This implies that, whenever we see in the data non-credible countries pegging their currency to a stable anchor, their inflation rates should drop (in line with the work from Levy-Yeyati and Sturzenegger (2010)) and their traded good consumption (which is part of GDP) should rise. The estimated values for credibility also suggest that even Germany gets more credible when Italy pegs its currency. The model is not able to explain this phenomenon in a unilateral peg, where Germany still does monetary policy only for itself. It can however explain a rise in measured credibility when a currency union is formed. A union-wide central bank that is as credible as the German central bank will conduct monetary policy for the whole union, which makes the policy less erratic as the central bank only reacts to average shock and not country specific shocks. In the data, this would mean that inflation is overall less volatile and also a bit lower on average, which is reflected in higher measured credibility. In the next section, we provide more empirical evidence on the effect of shifts in the exchange rate regime and inflation.

5 Empirical analysis

In this section, we start by describing the details of the global data set that we compiled for our analysis. Besides presenting the data sources, we provide a set of descriptive statistics together with an event study focusing on the dynamics of inflation, GDP and interest rates before and after a change in the exchange rate regime. Then, to test the implications of our model, we present our econometric analysis where we use an inverse probability weighted estimator to address the identification challenge present in our analysis - not all changes in the exchange rate regime are unexpected or exogenous to the business cycle. Finally, we present the empirical results based on the latter approach where we evaluate the impact of a pegging and a floating episode separately.

5.1 Data

We base our analysis on an unbalanced panel with annual data for 169 economies, including both Advanced Economies (AEs) and Emerging Economies (EMEs) over the last 70 years. The data used in this paper mainly relies on two sources: the IMF International Financial Statistics (IFS) database and the Penn World Table version 10.0 (Feenstra et al., 2015). We then complement these two datasets with information from the Macrohistory Database (Jordà et al., 2017).

We assemble data on the consumer price index inflation, short- and long-term interest rates, real gross domestic product growth rates, and bilateral exchange rates of the sample countries to the main anchor currencies in our sample: the US dollar, the German Mark and the British Pound.\(^8\)

We further complement the resulting dataset with the exchange rate regime classification from Ilzetzki et al. (2019). They identify the exchange rate regime in place for all countries in our sample based on both \textit{de jure} and \textit{de facto} classifications. Throughout the study, we will rely on their coarse episode classification which arguably identifies significant changes in the regime while further differentiating between a union and a peg regime.

In order to perform a consistent analysis, throughout the rest of the empirical analysis, we only use observations for which we have data on the exchange rate regime, CPI inflation and real GDP growth, rendering a total of 7,505 country-year observations between 1951 and 2016.\(^9\)

The reason why we do not limit ourselves to use observations for which we have information on short-term interest rates and exchange rates is data availability, if we restricted the sample in such a way we would end up with roughly 3,000 country-year observations, less than 50% of the original sample size. Nevertheless, we do so every time we evaluate the response of such variable or include it as a control variable.

### 5.1.1 Descriptive Statistics

Table 8: Summary Statistics (unweighted)

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<tr>
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<th>Float std. dev.</th>
<th>Peg mean</th>
<th>Peg std. dev.</th>
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<th>Union std. dev.</th>
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<td>2258</td>
<td></td>
<td></td>
<td>1211</td>
<td></td>
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<tr>
<td>gdp</td>
<td>4.18</td>
<td>4.78</td>
<td>4.57</td>
<td>4.81</td>
<td>3.74</td>
<td>6.09</td>
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<td>2258</td>
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<td></td>
<td>1211</td>
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<tr>
<td>Bills</td>
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<td>2.78</td>
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<td></td>
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<tr>
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<td>2.08</td>
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<tr>
<td>Obs</td>
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<td>593</td>
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<td>271</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the mean, within standard deviation and number of observations of each variable in our sample divided by exchange rate regime. According to Ilzetzki et al. (2019) classification, the Union columns comprise countries with no separate legal tender or in currency union. The Peg columns comprise countries in either a pre-announced peg or currency board arrangement, a pre-announced horizontal band that is narrower than or equal to +/-2%, or a de facto peg. Finally, the Float column comprises countries in all remaining exchange rate regimes. We only consider country-year observations for which inflation rates were below 99%.

Table 8 reports summary statistics of the key macroeconomics variables.\(^{10}\) We can high-

\(^{8}\)More information on the definition and source of these and other variables can be found in Appendix, Table A.1.

\(^{9}\)In Appendix, Figure A.1 gives more details about our sample coverage including the number of episodes by country.

\(^{10}\)Table A.2 in the Appendix provides further summary statistics when weighting the importance of a country by its population size. Table A.3 in the Appendix gives us the average duration in years of each of the identified exchange rate regimes.
light three stylized facts from the literature that are summarized in this Table: 1) inflation is higher and more volatile in floats than in pegs; 2) real GDP growth has similar behaviour; 3) interest rates are higher and more volatile in floats than in pegs.

Figure 2: Frequency of flexible and fixed regime changes

![Figure 2: Frequency of flexible and fixed regime changes](image)

Notes: Number of the changes of the exchange rate regime classification from Ilzetzki et al. (2019). Green bars: Move into a currency union or a no separate legal tender ($N = 23$). Orange bars: Move towards a peg regime ($N = 259$). Blue bars: Move towards a float' regime ($N = 266$). Figure A.2 in the Appendix further decomposes this graph between advanced and developing economies.

In a first step, let us look at the events in our sample where exchange rate regimes changed. Figure 2 illustrates how many times countries moved towards a more pegged or flexible regime over time. In our sample, we observe 259 pegging episodes and 266 floating episodes.

There are two big waves of regime adjustment episodes: a) following the Bretton Woods collapse in 1971, pegged countries were forced to float their currency or peg it to another anchor currency; and b) after 1990 there was a surge on pegging episodes (green bars) preceding both the Euro creation and the dollarization of emerging economies. Such variation is important to motivate our analysis.

5.1.2 Event Study

In the spirit of Eichengreen and Rose (2012), we now revisit our data and perform an event study exercise in order to analyze how key economic variables varied before and after a change in the exchange rate regime. Table 9 summarizes inflation, real GDP growth, short- and long-term interest rates before and after an episode where countries change their exchange rate regime, for the cross-section of countries in our sample that went through at least one such episode.\footnote{For completeness, we present the event study figures for this exercise in Appendix, Figures A.3, A.4, and A.5.}
Table 9: Event Study

<table>
<thead>
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<th></th>
<th>Float</th>
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</thead>
<tbody>
<tr>
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<td>mean</td>
<td>std. dev.</td>
<td>mean</td>
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<tr>
<td>inflation</td>
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<td>9.42</td>
</tr>
<tr>
<td>gdp</td>
<td>4.19</td>
<td>4.25</td>
<td>4.85</td>
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<td>Bills</td>
<td>9.99</td>
<td>12.80</td>
<td>4.95</td>
</tr>
<tr>
<td>Bond</td>
<td>8.49</td>
<td>9.34</td>
<td>2.52</td>
</tr>
</tbody>
</table>

Notes: This table presents both the mean and the within standard deviation of the four macroeconomics series (inflation, real GDP growth, short-term and long-term interest rates) before and after joining one of the three identified exchange rate regimes, according to Ilzetzki et al. (2019) classification. We only consider country-year observations for which inflation rates were below 99%.

Table 9 establishes three main observations. First, on average, inflation and interest rates decrease (increase) after a pegging (floating) episode. Second, the variability of inflation and interest rates goes down (up) under a peg (float), as the within country standard deviation gets smaller (larger). Third, we also find that real GDP slightly increases when the currency gets pegged. It is worth emphasizing that we derive symmetric observations between a pegging and floating episode for most variables except real GDP growth.

5.2 Econometric model

In this section, we provide reduced form evidence on the impact of changing the exchange rate regime. We base our empirical analysis on the same dataset and ask what is the impact of a change in the exchange rate regime on inflation, real GDP growth, and, in a second step, interest rates.

To estimate the impact of changing the exchange rate regime (ERR), we need to compare two counterfactual scenarios: One where the representative country in our sample effectively changed the ERR and the other where it did not. If the ERR change decision was random, it would be sufficient to compare the average performance of changers to non-changers. However, we know that most countries do not randomly decide to change their ERR.

Historically, there are two well studied episodes that offer quasi-random variation. First, the United States’ unilateral decision of terminating the convertibility of the US dollar to gold on 15 August 1971. This event effectively led to the collapse of the Bretton Woods agreement, and thus forced countries to change their exchange rate regime (Bordo and Eichengreen, 2019). While some were forced to immediately float their currency, others decided to peg to another anchor currency, with the German Mark being one of the preferred currencies (Ilzetzki et al., 2019).

The second episode was the Euro creation. Eurozone accession was driven mainly by political rather than economic factors (Feldstein, 1997). In fact, by not satisfying the requirements
of an Optimum Currency Area, many economists believed that countries adopting the Euro would face economic losses (Jonung et al., 2009), belief that was later corroborated in recent works by Puzzello and Gomis-Porqueras (2018) and Gabriel and Pessoa (2020). Notwithstanding, it is not true that all such events in our sample are as good as random.

We thus accept that some changes in the ERR decisions in our dataset are more endogenous than others, but we seek to explicitly model this endogenous decision process and account for it in our estimation. By modelling the ERR change decision, we can effectively reverse-engineer it and rebalance the sample “as if” it was taken at random. To do this, we use the inverse propensity score weighting methodology cleanly exposed in Caliendo and Kopeinig (2008).

5.2.1 Methodology

It is possible that policymakers choose a specific exchange rate regime due to current economic circumstances or because they wanted to achieve a certain economic outcome such as lower inflation. Those changes in the exchange rate regime cannot be seen as exogenous and are hence uninformative in inferring causal effects of a fixed or a flexible regime.

To estimate the causal response, we thus employ an inverse probability weighted regression-adjusted (IPWRA) estimator. An inverse probability weighted (IPW) estimator gives more weight to those events that are difficult to predict based on observable macroeconomic variables and less weight to those instances that are endogenous due to the other factors. This estimator will thus rebalance the sample to mimic a setting where the ERR change decision was random. Applications of such method study not only the effect of changes in the ERR on the foreign direct investment (Cushman and De Vita, 2017), but also other topics such as the economic response to austerity (Jordà and Taylor, 2016), sovereign defaults (Kuvshinov and Zimmermann, 2019), and macroprudential policy changes (Richter et al., 2019). We will follow the notation established in the latter work throughout the rest of the empirical section.

Let \( d_{i,t} \) be a dummy variable that takes value 1 if there was a change in the exchange rate regime and zero otherwise. The estimation proceeds in two stages. In the first stage, we model the ERR change decision by estimating a propensity score for each observation in our sample. Such score is obtained by a logit model which estimates the probability that the ERR is going to change as follows:

\[
\log \left( \frac{P[d_{i,t} = 1|Z_{i,t-1}]}{P[d_{i,t} = 0|Z_{i,t-1}]} \right) = \alpha_i + \beta Z_{i,t-1} + \varepsilon_{i,t} \tag{4}
\]

where \( Z_{i,t-k} \) is a vector of macroeconomic controls at time \( t-1 \) and \( t-2 \) with the purpose of controlling for business cycle fluctuations, where we include the lagged growth rates of real
GDP, trade openness, government spending, and CPI inflation. Growth rates are computed as log differences to avoid results being driven by extreme values. Moreover, we exclude observations where lagged absolute values of inflation or real GDP growth was above 100%. In addition, we include country fixed effects to account for country-specific trends. We refer to the probability of a tightening as the propensity score and its estimate from Equation (4) is denoted by \( p_{i,t} \). We report results using logit but using probit made very little difference to the results of a number of cases where we tried it, consistent with the discussion in Caliendo and Kopeinig (2008).

In the second stage, we estimate local projections using regression weights given by the inverse of \( p_{i,t} \). To be precise, the weights are defined by \( w_{i,t} = \frac{d_{i,t}}{p_{i,t}} + \frac{1 - d_{i,t}}{1 - p_{i,t}} \), where we truncate \( w_{i,t} \) at 10. Weighting by the inverse of the propensity score puts more weight on those observations that were difficult to predict and thereby re-randomises the treatment. In our application, this implies putting more weight on exchange rate regime changes that were taken as a surprise based on observable macroeconomic variables, and putting less weight on those changes that could be predicted. For example, for the evaluation of the impact of pegging a currency we are giving more weight to the de facto peg to the Deutsche Mark by Spain in 1994, and less weight to the de facto peg to the US dollar by Ukraine in 2000.

Once the sample is rebalanced, the impact of an ERR change is measured as its “average treatment effect”, that is, the average difference in potential outcomes of changers and non-changers across the sample. Potential outcomes are computed using a conditional local projection forecast over a horizon of 5 years (Jordà, 2005). To implement the second stage, we thus run the following specification using weighted least squares:

\[
\Delta_h y_{i,t+h} = \alpha_i^h + \gamma_t^h + \Gamma_h d_{i,t} + \phi_h Z_{i,t-k} + \epsilon_{i,t+h}, \quad h \in \{0, \ldots, 6\}
\] (5)

where \( \Delta_h y_{i,t+h} = \ln(y_{i,t+h}) - \ln(y_{i,t-1}) \) is the conditional forecast of the cumulative growth in percent in one of the outcome variables (real GDP or inflation), in country \( i \) between base year \( t - 1 \) and year \( t + h \) over varying prediction horizons \( h = 0, 1, \ldots, 5 \) years. \( d_{i,t} \) is the treatment dummy variable as before, taking a value of 1 whenever there is a pegging (or floating) episode and thus \( \Gamma^h \) is our coefficient of interest.

We include a rich set of covariates in each specification including country dummies to control for country-specific growth rates \( \alpha_i^h \) as well as time-fixed effects \( \gamma_t^h \) to control for global trends. Moreover, we include \( Z_{i,t-k} \) which is a vector consisting of 2 lags real GDP growth, inflation, trade openness, and a regime dummy, just like in equation 4. Finally, \( \epsilon_{i,t+h} \) is the

---

12The choice of control variables follows the work of Poirson (2001). However, in order to keep the number of studied episodes and, consequently, observations at the maximum possible level, we will only include other important control variables like the short- and long-term interest rates, and the standard deviation of the 12 months exchange rate against the US dollar in the robustness section.
error term, and the standard errors are clustered by country. This procedure assigns a higher weight to the treated observations that were less likely to be treated based on this analysis, i.e. those observations with very low probabilities. Further details on the methodology can be found in Jordà and Taylor (2016).

5.2.2 Econometric results

Figures 3 and 4 present the main results. To put our findings in perspective, we estimate Equation (5) using both WLS and OLS. This way we can evaluate the correction of the expected bias.

Figure 3: IPWRA Results of a pegging event

![IPWRA Results of a pegging event](image)

(a) CPI inflation  (b) Real GDP growth

Notes: The figure shows the impulse response functions for inflation and real GDP growth in percent over time, when the exchange rate regime becomes more pegged. Equation (5) has been estimated with weighted least squares. The weights correspond to the inverse estimated probability of an exchange rate regime change from (4). The (dark) gray shaded areas indicate a confidence interval of (68%) 90%. The black dashed line shows the OLS estimates.

The estimates suggest that pegging episodes seem to have significant and persistent effects on inflation and real GDP growth. If a country moves towards a more pegged exchange rate regime, inflation goes persistently down by up to 15%. In the analyzed sample, mean inflation is around 16%, meaning that a pegging episode would, on average, diminish inflation by 2.5 percentage points. Finally, real GDP reacts positively growing 1% on impact with its cumulative growth becoming 3% at horizon 5.

In contrast, a shift towards a more floating regime, leads to a strong and significant increase in both inflation while the GDP response is negative on impact with both becoming insignificant in the medium-term, thus pointing to a less persistent impact of a floating episode.
6 Conclusion

In this paper, we reassess the gains of adopting a fixed exchange rate regime. The underlying problem with a flexible exchange rate regime is that independent central banks with a lack of credibility tend to use surprise inflation to promote economic growth. This surprise inflation leads to an inflationary bias that increases the level of inflation and interest rates persistently. Entering a fixed exchange rate regime helps to mitigate this commitment problem and not only reduces inflation but also promotes economic growth persistently.

Our contribution is thus two-folded. First, we provide an estimated version of the model by Chari et al. (2020a) to show that countries displaying such an inflationary bias can solve such bias by giving up monetary autonomy and pegging the exchange rate to a stable anchor country. We simulate the model under different exchange rate regimes under both commitment and discretion and derive two key implications: inflation (real GDP) decreases (increases) persistently after adopting a fixed exchange rate regime.

Second, we test and corroborate these implications by making use of a comprehensive annual macroeconomics dataset covering 169 countries over the last 70 years together with an inverse probability weighted estimator. We show that, on average, inflation goes persistently down by up to 15% while real GDP displays a positive cumulative growth of 3% 5 years after the change.

For future research, we plan to test the credibility channel in the data by first classifying countries in credible or non-credible based on different indicators such as inflation and exchange rate volatility and institutions’ quality, and test whether there are different responses
to changes in these countries’ exchange rate regimes. Furthermore, it would also be interesting to test the Neo-Fisher effect and see whether the response of nominal interest rates are closely aligned with the response of inflation.
References

choices and consequences, volume 1. MIT press.


Appendix A  Data and other empirical findings

Table A.1: Variables Description

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
<th>Source</th>
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<tr>
<td>inflation</td>
<td>Consumer Price Index Growth Rate (in p.p.)</td>
<td>IMF IFS</td>
</tr>
<tr>
<td>gdp</td>
<td>Gross Domestic Product Real Growth Rate (in %)</td>
<td>IMF IFS</td>
</tr>
<tr>
<td>Bills</td>
<td>Government Bond Yields, Short to Medium-Term, Percent per Annum [FIGBY_SM_PA], when needed complemented with Interest Rates, Government Securities, Treasury Bills, Percent per annum [FITB_PA], and Interest Rates, Government Securities, Treasury Bills, 3 month, Percent per annum [FITB_3M_PA]</td>
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<td>Bond</td>
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<td>ERA</td>
<td>Exchange Rate Agreement</td>
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</tr>
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</table>
Figure A.1: Episodes and Data Coverage

Notes: Number of the changes of the exchange rate regime classification towards a more Peg or Float regime from Ilzetzki et al. (2019). Data coverage for each country, begin and end of sample, for which we have information on the exchange rate regime classification, inflation, and real GDP growth rate.

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Notes: Number of the changes of the exchange rate regime classification towards a more Peg or Float regime from Ilzetzki et al. (2019). Data coverage for each country, begin and end of sample, for which we have information on the exchange rate regime classification, inflation, and real GDP growth rate.
Table A.2: Summary Statistics (weighted by population size)

<table>
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<td></td>
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<td>mean</td>
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<td>Obs</td>
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Notes: This table reports the mean, within standard deviation and number of observations of each variable for our sample divided by exchange rate regime and weighted by the population size. According to Ilzetzki et al. (2019) classification, the Union columns comprise countries with no separate legal tender or in currency union. The Peg columns comprise countries in either a pre-announced peg or currency board arrangement, a pre-announced horizontal band that is narrower than or equal to +/-2%, or a de facto peg. Finally, the Float columns comprise countries in all remaining exchange rate regimes.

Table A.3: Average Duration of Exchange Rate Regimes

<table>
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<tr>
<td>Average Duration (years)</td>
<td>22.8</td>
<td>15.1</td>
<td>23.8</td>
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</table>

Notes: This table reports the average duration of each exchange rate regime in years in our sample, according to the Ilzetzki et al. (2019) classification.

Figure A.2: Frequency of flexible and fixed regime changes

Notes: Number of the changes of the exchange rate regime classification from Ilzetzki et al. (2019). Orange bars: Move towards a peg regime (N = 259). Blue bars: Move towards a float regime (N = 266).
Figure A.3: for a pegging episode

(a) Inflation  
(b) Short-term interest rate  
(c) Real GDP growth  
(d) Long-term interest rate

Notes: The figure shows the event-study for median inflation and median interest rates in percentage points, and median real GDP growth in percent before and after a pegging episode, when the exchange rate regime becomes more pegged.

Figure A.4: Event-study for a floating episode

(a) inflation  
(b) short-term interest rate  
(c) real GDP growth  
(d) long-term interest rate

Notes: The figure shows the event-study for median inflation and median interest rates in percentage points, and median real GDP growth in percent before and after a floating episode, when the exchange rate regime becomes more float.

Figure A.5: Event-study for a union episode

(a) inflation  
(b) short-term interest rate  
(c) real GDP growth  
(d) long-term interest rate

Notes: The figure shows the event-study for median inflation and median interest rates in percentage points, and median real GDP growth in percent before and after a union episode, when countries enter in a currency union.
Figure A.6: IPWRA Results of a union event

(a) CPI inflation

(b) Real GDP growth

Notes: The figure shows the impulse response functions for inflation and real GDP growth in percent over time, when the exchange rate regime becomes a union. Equation (5) has been estimated with weighted least squares. The weights correspond to the inverse estimated probability of an exchange rate regime change from (4). The (dark) gray shaded areas indicate a confidence interval of (68%) 90%. The black dashed line shows the OLS estimates.
A.1 Case Study: Italy and Germany (and other countries)

We start by providing descriptive evidence about the relationship between the exchange rate regime and nominal macroeconomic variables in Italy (as the pegging country) and Germany (as the anchor country). We take Germany as the benchmark because it is the largest economy in Europe and plays a pronounced role for the continent’s economy. With this assessment, we follow Ilzetzki et al. (2019) who identify Germany as the anchor country for most continental (western) European countries following the breakdown of the Bretton Woods agreement.

Figure A.7a shows the bilateral exchange rate of the Italian Lira to the German Mark between 1954 and 2016. The exchange rate is indexed to 1 in 1955, the data are taken from the Bundesbank. Figure A.7b shows the inflation rate of Italy and Germany. The exchange rate regime changes are identified by a vertical blue (peg) and red (float) lines:

Figure A.7: Exchange rate and inflation in Italy

(a) Exchange Rate (1955=1) 
(b) Inflation (in %)

Notes: Graph (a) shows the evolution of the bilateral exchange rate of the Italian Lira to the German Mark normalized to 1 in 1955. Graph (b) shows how inflation in Germany (dashed red line) and Italy (dotted green line) co-moved over time. According to the fine classification of Ilzetzki et al. (2019), the vertical red lines indicate a fall of the exchange rate or a shift towards a floating exchange rate regime, the blue vertical lines a shift towards an exchange rate regime that is more pegged and that was followed by a stabilization of the exchange rate. Sources: Bundesbank, IFS, and Ilzetzki et al. (2019).

At the beginning of our sample Italy and Germany were both in a fixed exchange rate regime. There is almost no movement in the exchange rate and inflation moves below 5% for both countries. After the end of Bretton Woods, Italy’s currency experiences a large depreciation. This coincides with a large increase of Italy’s inflation. Inflation peaks at over 20 percent after 1980. After 1985, as the exchange rate gets pegged to the German Mark, the behavior of Italy’s inflation changes: Fixing the exchange rate to Germany coincides with a convergence of inflation to the relatively low and stable German level.

Moreover, there seems to be a change in the behavior of the variability of inflation: During the time of a flexible exchange rate regime - between 1972 and 1985 - inflation displayed higher volatility. In contrast, volatility decreased from the 90’s onward, marking the arrival of the Euro. This decline in volatility was very pronounced for Italy, but also clearly visible for Germany. Furthermore, when comparing Germany’s inflation during the episodes of flexible
exchange rates with those episodes with fixed exchange rates, it seems that average inflation is also slightly lower with fixed rates.

Other southern European countries, like Spain or Portugal (Figure A.8), experienced similar patterns: A stable exchange rate to the German Mark coincided with similar inflation rates, but when monetary policy was conducted independently without any exchange rate goal, the exchange rate depreciated, inflation substantially increased compared to Germany and the variability went up. Contrarily, countries like Austria and the Netherlands had their inflation closely tracking Germany’s inflation (Figure A.9).

Figure A.8: Exchange rate and inflation in Spain and Portugal

Notes: Graph (a) shows the evolution of the bilateral exchange rate of the Spanish currency to the German Mark normalized to 1 in 1955. Graph (b) shows how inflation in Germany (dashed red line) and the Spain (dotted green line) co-moved over time. Inflation and Exchange rate Portugal Graph (c) shows the evolution of the bilateral exchange rate of the Portuguese currency to the German Mark normalized to 1 in 1955. Graph (d) shows how inflation in Germany (dashed red line) and the Portugal (dotted green line) co-moved over time.
Figure A.9: Exchange rate and inflation in Austria and the Netherlands

(a) Exchange Rate AUT-DEU (1955=1)

(b) Inflation (in %)

(c) Exchange Rate NLD-DEU (1955=1)

(d) Inflation (in %)

Notes: Graph (a) shows the evolution of the bilateral exchange rate of the Austrian Schilling to the German Mark normalized to 1 in 1955. Graph (b) shows how inflation in Germany (dashed red line) and Austria (dotted green line) co-moved over time. The vertical red lines indicate a fall of the exchange rate or a shift towards a floating exchange rate regime, the blue vertical lines a shift towards an exchange rate regime that is more pegged and that was followed by a stabilization of the exchange rate. Inflation and Exchange rate Netherlands Graph (c) shows the evolution of the bilateral exchange rate of the Dutch currency to the German Mark normalized to 1 in 1955. Graph (d) shows how inflation in Germany (dashed red line) and the Netherlands (dotted green line) co-moved over time. Sources: Bundesbank and IFS
Appendix B  Model

B.1 Consumer Optimization

The Lagrangean is

\[
\max_{C_T, C_N, L, B^*, M_H} \mathcal{L} = \sum_{t=0}^{\infty} \sum_{s'} \beta^t h_t \left( s' \right) \left[ \alpha \log C_T \left( s' \right) + \left( 1 - \alpha \right) \log C_N \left( s' \right) - \psi L \left( s' \right) - \lambda \left( s' \right) \left( P_T \left( s' \right) C_T \left( s' \right) + P_N \left( s'-1, s_{1t} \right) C_N \left( s' \right) + M_H \left( s' \right) + B \left( s' \right) + e \left( s' \right) B^* \left( s' \right) - \left( P_T \left( s' \right) L \left( s' \right) + M_H \left( s'-1 \right) + R \left( s' \right) B \left( s'-1 \right) + e \left( s' \right) R^* \left( s' \right) B^* \left( s'-1 \right) + T \left( s' \right) + \Pi \left( s' \right) \right) \right] - \mu \left( s' \right) \left( P_T(s') C_T(s') - M_H(s'-1) \right) \right] \]

The first order conditions are

\[
\frac{\alpha}{C_T(s')} = \lambda(s') P_T(s') + \mu(s') P_T(s') \tag{B.1}
\]
\[
\frac{1 - \alpha}{C_N(s')} = \lambda(s') P_N(s') \tag{B.2}
\]
\[
\psi = \lambda(s') P_T(s') \tag{B.3}
\]
\[
\lambda(s') = \beta \mathbb{E}_t \left[ \lambda(s'^{t+1}) R \left( s'^{t+1} \right) \right] \tag{B.4}
\]
\[
\lambda(s') e(s') = \beta \mathbb{E}_t \left[ \lambda(s'^{t+1}) e \left( s'^{t+1} \right) R^* \left( s'^{t+1} \right) \right] \tag{B.5}
\]
\[
\lambda(s') = \beta \mathbb{E}_t \left[ \lambda(s'^{t+1}) \right] + \beta \mathbb{E}_t \left[ \phi \left( s'^{t+1} \right) \right] \tag{B.6}
\]

Combining (B.2) and (B.4) gives the Euler equation:

\[
\frac{1}{C_N(s')} = \beta \mathbb{E}_t \left[ \frac{1}{C_N(s'^{t+1})} \frac{P_N(s')}{P_N(s'^{t+1})} R(s'^{t+1}) \right] \]

Combining (B.4) and(B.5) gives the uncovered interest parity condition:

\[
\beta \mathbb{E}_t \left[ \lambda(s'^{t+1}) R \left( s'^{t+1} \right) \right] = \beta \mathbb{E}_t \left[ \lambda(s'^{t+1}) \frac{e \left( s'^{t+1} \right)}{e \left( s' \right)} R^* \left( s'^{t+1} \right) \right] \]

The standardized multiplier on the cash in advance constraint is \( \phi(s') = \mu(s') P_T(s') \).

If we use Chari et al. (2020a) framework of prices on bonds instead of interest rates, the budget constraint changes to

\[
P_T \left( s' \right) C_T \left( s' \right) + P_N \left( s'-1, s_{1t} \right) C_N \left( s' \right) + M_H \left( s' \right) + Q(s') B \left( s' \right) + \bar{Q}(s') e \left( s' \right) B^* \left( s' \right) \leq P_T \left( s' \right) L \left( s' \right) + M_H \left( s'-1 \right) + B \left( s'-1 \right) + e \left( s' \right) B^* \left( s'-1 \right) + T \left( s' \right) + \Pi \left( s' \right) \]
The first order condition is then
\[ \lambda(s^t) = \beta E_t \left[ \lambda(s^{t+1}) \frac{1}{Q(s^t)} \frac{\lambda(s^t)}{R(s^{t+1})} \right] \]

So, using a framework with bond prices instead of interest rates on one period government bonds means that the price of a new bond is the inverse nominal interest rate on bonds that are being hold. \( R(s^{t+1}) \) is known in \( t \).

**B.2 International Capital Markets**

The budget constraint is extended to allow households to buy non-domestic bonds as well. These bonds \( B^* \) are risk free and denoted in foreign currency:

\[
P_T(s^t) C_T(s^t) + P_N(s^{t-1}, s_{1t}) C_N(s^t) + M_H(s^t) + Q(s^t) B^*(s^t) + \bar{Q}(s^t) \lambda(s^t) = P_T(s^t) L(s^t) + M_H(s^{t-1}) + B(s^{t-1}) + e(s^t) B^*(s^{t-1}) + \Pi(s^t)
\]

(B.7)

The exchange rate \( e(s^t) \) has to be taken into account. As households can now choose non-domestic bonds, there is a a new first order condition:

\[ \bar{Q}^*(s^t) \lambda(s^t) e(s^t) = \beta E_t \left[ \lambda(s^{t+1}) e(s^{t+1}) \right] \]

Combining it with the old conditions

\[ \bar{Q}(s^t) \lambda(s^t) = \beta E_t \left[ \lambda(s^{t+1}) \right] \]

\[ \lambda(s^t) = \frac{\alpha}{P_T(s^t) C(s^t)} \]

gives the so-called uncovered interest rate parity that relates domestic with foreign interest rates:

\[ \frac{\bar{Q}^*(s^t) e(s^t)}{Q(s^t)} = \frac{E_t \left[ \lambda(s^{t+1}) e(s^{t+1}) \right]}{E_t \left[ \lambda(s^{t+1}) \right]} \]

with iid shocks we have

\[ E_t \left[ Q(s^{t+1}) R(s^{t+1}) \right] = E_t \left[ Q(s^{t+1}) \frac{e(s^{t+1})}{e(s^t)} R^*(s^{t+1}) \right] \]

The nominal interest rate spread is offset by a continuous devaluation of the home currency vis-a-vis to the rest of the world. The rest of the model is not altered by the introduction of international capital markets, as households do not have an incentive to borrow or lend across countries given their current preference structure (log utility and Cobb Douglas consumption aggregator).
B.3 Firm Optimization

A microfoundation for markups can be given by following the setup of Smets and Wouters (2007). The non-traded good is produced by a competitive final producer who uses differentiated inputs \( y_N(j, s^t) \) from input firms of mass \( j \in [0, 1] \) to produce the final good \( Y_N(s^t) \):

\[
Y_N(s^t) = \left[ \int y_N(j, s^t)^{\theta(s_{1t})} \, dj \right]^{1/\theta(s_{1t})}
\]

This firm maximizes

\[
P_N(s^{t-1}, s_{1t}) Y_N(s^t) - \int P_N(j, s^{t-1}, s_{1t}) y_N(j, s^t) \, dj
\]

Demand for intermediate goods is therefore

\[
y_N(j, s^t) = \left( \frac{P_N(s^{t-1}, s_{1t})}{P_N(j, s^{t-1}, s_{1t})} \right)^{1/\theta(s_{1t})} Y_N(s^t)
\]

Intermediate goods are produced by monopolistic firms who use a linear production function: \( y_N(j, s^t) = A(s_{2t}) L_N(j, s^t) \). Intermediate good firms choose their prices \( P = P(j, s^{t-1}, s_{1t}) \) to maximize their profits:

\[
\max_P \sum_{s_{2t}} Q(s^t) \left[ P - \frac{W(s^t)}{A(s_{2t})} \right] \left( \frac{P_N(s^{t-1}, s_{1t})}{P} \right)^{1/\theta(s_{1t})} Y_N(s^t)
\]

where \( Q(s^t) \) is the discount factor as before. We assume that \( \theta(s_{1t} \in (0, 1) \) implying elastic demand and finite prices. Optimally, intermediate good producer \( j \) sets the price in the following way:

\[
P_N(j, s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t})} \sum_{s_{2t}} Q(s^t) Y_N(s^t) \frac{W(s^t)}{A(s_{2t})}
\]

Where \( \frac{1}{\theta(s_{1t})} \) is the markup that increases prices. Note that the price equation is not a function of \( j \) such that the price is the same for all intermediate firms. Plugging in \( W(s^t) = P_T(s^t) \) gives the same formula as in equation (1). This implies that all intermediate firms hire the same amount of labor and their production function is then simply \( Y_N(s^t) = A(s_{2t}) L_N(s^t) \).
B.4 Monetary Policy Optimization

Commitment and Float  The central bank makes a state-contingent plan for prices of traded and non traded goods to maximize the representative households ex ante utility

\[
\max_{\{p_T(s^t), p_N(s^t)\}_{t=0}^\infty} \mathbb{E}_0 \left[ \sum_{t=t} \beta^t \left( \alpha \log(C_T(s^t)) + (1 - \alpha) \log(C_N(s^t)) - \psi L(s^t) \right) \right]
\]

s.t.  
\[
L(s^t) = \frac{C_N(s^t)}{A(s^t_{2t})} + C_T(s^t) = \frac{1 - \alpha}{\psi} \frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})}
\]

\[
C_T(s^t) = \frac{\alpha}{\psi}
\]

\[
C_N(s^t) = \frac{1 - \alpha}{\psi} \frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})}
\]

\[
\sum_{s_{2t}} h(s^t | s^{t-1}, s_{1t}) C_N(s^t) \left[ \frac{1 - \alpha}{C_N(s^t)} - \frac{1}{\theta(s_{1t})} \right] = 0
\]

Looking at the plugged in firm’s first order condition:

\[
\sum_{s_{2t}} h(s^t | s^{t-1}, s_{1t}) C_N(s^t) \left[ \frac{1 - \alpha}{C_N(s^t)} - \frac{1}{\theta(s_{1t})} \right] = 0
\]

Plugging in \( C_N \)

\[
\sum_{s_{2t}} h(s^t | s^{t-1}, s_{1t}) C_N(s^t) \left[ \frac{1 - \alpha}{C_N(s^t)} - \frac{1}{\theta(s_{1t})} \right] = 0
\]

This can only be zero if

\[
\frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})} = A(s_{2t}) \theta(s_{1t})
\]

The best the central bank can do is to ensure that this condition holds. The central bank realizes that it is not possible to reduce markups by manipulating relative prices with inflation. Therefore it focuses to stabilize productivity shocks.

Nominal variables can be computed as well, using the following trick: First normalize all variables with their money supply of the last period, \( p_T = \frac{P_T(s^t)}{M(s^{t-1})} \) and \( p_N(s^{t-1}, s_{1t}) = \frac{P_N(s^{t-1}, s_{1t})}{M(s^{t-1})} \). Then construct prices in such a way, that the cash in advance constraint is exactly binding in the highest possible productivity state\(^{13}\). Then use that \( \frac{p_T(s^t)}{p_N(s^{t-1}, s_{1t})} = A(s_{2t}) \theta(s_{1t}) \) and \( p_T(s^t) = \gamma C_T(s^t)^{-1} \) if the cash in advance constraint binds in the highest

\(^{13}\)This way, no consumption is lost.
state to get:

\[ p_N(s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t}) \alpha \max\{A(s_{2t})\}} \]

\[ p_T(s^t) = A(s_{2t}) \theta(s_{1t}) p_N(s^{t-1}, s_{1t}) \]

\[ \frac{M(s^t)}{M(s^{t-1})} = \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{A(s_{2t})}{A(s_{2t+1})} \]

Together with an initial level for \( M(s^0) \), the nominal equilibrium is pinned down. The per period money growth rate equals productivity today times the discounted expected inverse productivity in the future. If productivity today is relatively large, money growth will also be relatively large, reflecting expansionary monetary policy and a depreciating exchange rate from the example before. If productivity is not stochastic, money gross growth rate is \( \beta < 1 \).

The derivation from the money growth rate comes from the consumer’s first order condition, that combines the labor and traded goods first order condition with the money first order condition. As \( p_T(s^t) = P_T(s^t)/M(s^{t-1}) \), we can draw out the money growth rate as follows

\[ -\frac{M(s^t)}{M(s^{t-1})} \frac{U_L}{p_T} = \beta \sum_{s'} h(s') \frac{U_T(b', 1, x'_T, S'_T)}{p_T(x'_T, S'_T)} \]

If you rearrange and plug in, you arrive at

\[ \frac{M(s^t)}{M(s^{t-1})} = \beta \sum_{s'} h(s') \frac{p_T(s)}{p_T(s')} \frac{\alpha/\psi}{C_T(s')} \]

Plugging in \( p_T = A\theta p_N \) with \( p_N = \frac{1}{\theta} \frac{1}{\alpha \max\{A\}} \) at a binding cash in advance constraint with \( C_T(s') = \frac{1}{p_T(s')} \) gives the money growth rate as above, only a function of productivity.

Nominal interest rates can then be computed via the Euler equation, see Appendix B.6 for a derivation

\[ R(s^t) = \max\{A(s_{2t})\}/\max\{A(s_{2t+1})\} \]

Interest rates are simply the ratio of the maximum value of productivity today and tomorrow. If productivity stays always the same, then \( R(s^t) = 1 \) and \( M(s^t)/M(s^{t-1}) = \beta < 1 \). This means that nominal interest rates are zero and the central bank follows the Friedman rule implying a negative money growth rate. The intuition why zero interest rates are optimal is the following. Nominal bonds dominate money holding as they pay an interest on its stock, while money does not. Nevertheless, households need to hold money to buy traded goods. Therefore, the central bank optimally implements zero interest rates to make the necessary
money growth rate which is then simply $\beta < 0$. Both inflation rates are given by

$$\pi_N(s^t, s_{t+1}) = \frac{P_N(s^t, s_{t+1})}{P_N(s^{t-1}, s_t)} = \frac{\theta(s_{t+1}) \max\{A(s_{2t})\} M(s^t)}{\theta(s_t) \max\{A(s_{2t+1})\} M(s^{t-1})}$$

$$\pi_T(s^{t+1}) = \frac{P_T(s^{t+1})}{P_T(s^t)} = \frac{A(s^{t+1})\theta(s_{t+1})P_N(s^t, s_{t+1})}{A(s^t)\theta(s_{t+1})P_N(s^{t-1}, s_t)} = \frac{\theta(s_{t+1}) \max\{A(s_{2t+1})\} M(s^t)}{\theta(s_{t+1}) \max\{A(s_{2t+1})\} M(s^{t-1})}$$

Markups influence prices of non-traded goods only. The bigger the markup (1/$\theta$ is high) compared to last period, the higher is inflation. Higher productivity of the non-traded good increases prices of traded goods, the relative price adjusts. Higher money growth rates increase both inflation rates. In a world with no stochastic components, inflation is determined by the money growth rate which is then simply $\beta < 0$. This implies deflation. The Friedman rule is a solution for the nominal equilibrium, a continued contraction of the money supply implies deflation which ensures that the cash in advance constraint is never binding.

**Discretion and Float** Chari et al. (2020a) show, that there is no intertemporal dimension of the problem for the central bank. The reason is that in equilibrium there is no bond holding and that lump-sum transfers are always available to the government. In addition, households do not derive utility out of money, such that the growth rate of money supply is not intertwined with the static problem. The optimal problem of the central bank can then be thought of as choosing the price of the traded good subject to the first order conditions of households. As the cash in advance constraint optimally binds for the traded good, the primal problem of the central bank is to maximize

$$\max_{P_T(s^t)} \quad \alpha \ln C_T(s^t) + (1-\alpha) \ln C_N(s^t) - \psi(C_T(s^t) + C_N(s^t)/A(s^t))$$

subject to

$$C_T(s^t) = \frac{M(s^{t-1})}{P_T(s^t)}$$

$$C_N(s^t) = \frac{1 - \alpha}{\psi} \frac{P_T(s^t)}{P_N(s^t)}$$

The first order condition is (already divided by $M(s^{t-1})$

$$-\frac{\alpha}{p_T(s^t)} + \frac{1 - \alpha}{p_T(s^t)} - \psi \left( -\frac{1}{p_T(s^t)^2} + \frac{1 - \alpha}{\psi} \frac{1}{A(s^t)} \frac{1}{p_N(s^t)} \right) = 0$$

Solving for $p_T(s^t)$ gives the optimal reaction function of the central bank under discretion:

$$p_T(s_t) = p_N(s_{1t}) A(s_{2t}) \frac{1}{2(1-\alpha)} \left[ (1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)^2} \frac{\psi}{A(s_{2t}) p_N(s_{1t})} \right]$$

$$R\left(\pi(s_{2t}) s_{1t}\right)$$
If you consider the firm’s price setting equation 1, then you can compute prices:

\[
p_N(s_{t-1},s_{1t}) = \frac{1}{\theta(s_{1t})} \sum_{s_{2t}} \left( \frac{Q(s') Y_N(s')}{{\sum_{s_{2t}} Q(s') Y_N(s')}} \right) p_N(s_{1t}) A(s_{2t}) \frac{1}{2(1-\alpha)} \left[ (1 - 2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s_{2t}) p_N(s_{1t})}} \right] A(s_t)
\]

If \(p_N\) rises, \(p_T\) will then in general rise by less than 1 to 1, reflecting the costs of higher inflation.

If \(A\) is not stochastic the cash in advance constraint never binds (implicit assumption here).

We can then write

\[
1 = \frac{1}{\theta(s_{1t})} A(s_{2t}) \frac{1}{2(1-\alpha)} \left[ (1 - 2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s_{2t}) p_N(s_{1t})}} \right] A(s_t)
\]

Solving for \(p_N(s_{1t})\) gives

\[
(2(1-\alpha)\theta - (1 - 2\alpha))^2 = (1 - 2\alpha)^2 + 4(1-\alpha)\psi p_N(s_{1t})
\]

With this we get \(p_N\) as in the main text:

\[
p_N(s') = \frac{1}{\theta(s_{1t})} \frac{1}{A(s_{2t})} \frac{\psi}{\alpha - (1-\alpha)(1-\theta(s_{1t}))} p_T(s') = \frac{\psi}{\alpha - (1-\alpha)(1-\theta(s_{1t}))}
\]

Consumption is then

\[
C_T(s') = \frac{1}{p_T(s')} \quad C_N(s') = \frac{1 - \alpha}{\psi} \frac{p_T(s')}{p_N(s')}
\]

The money growth rate and inflation rates can be computed as before

\[
\frac{M(s')}{M(s^{t-1})} = \beta \sum_{s'} h(s') \frac{p_T(s)}{p_T(s')} \frac{\alpha/\psi}{1/p_T(s')}
\]

\[
\frac{M(s')}{M(s^{t-1})} = \beta \sum_{s'} h(s') \frac{p_T(s)}{p_T(s') \frac{\alpha/\psi}{1/p_T(s')}}
\]

\[
\frac{M(s')}{M(s^{t-1})} = \beta \sum_{s'} h(s') \frac{p_T(s)}{p_T(s') \frac{\alpha/\psi}{1/p_T(s')}}
\]

\[
\frac{M(s')}{M(s^{t-1})} = \beta \frac{\alpha}{\psi} \frac{1}{\alpha - (1-\alpha)(1-\theta(s'))}
\]
Inflation rates are then

\[ \pi_T(s^t) = \frac{P_T(s^{t+1})}{P_T(s^t)} = \frac{M(s^t)}{M(s^{t-1})} \alpha - (1 - \alpha)(1 - \theta(s_{1t})) \]

\[ \pi_N(s^t) = \frac{P_N(s^{t+1})}{P_N(s^t)} = \frac{M(s^t)A(s^t)\theta(s^t)}{M(s^{t-1})A(s^{t+1})\theta(s^{t+1})} \alpha - (1 - \alpha)(1 - \theta(s_{1t})) \]

**Unilateral Peg to a Credible Anchor** The central bank of the anchor country sets monetary policy as before

\[ \frac{P_{Anch}^T(s^t)}{P_{Anch}^N(s^{t-1}, s_{1t})} = A_{Anch}^T(s_{2t}) \theta_{Anch}(s_{1t}) \]

The client country’s central bank then ensures that the exchange rate is always constant such that the price of traded goods is exactly the same in both countries.

Firms of the client country, after their markup shock has realized then set prices the following way:

\[ p_N(s) = \frac{1}{\theta(s)}\mathbb{E}\left[ \frac{p_{T}^{Anch}(s)}{A_{Anch}(s)} \right] \]

For the client country the allocation of consumption is then given by

\[ C_T(s) = \min\{\frac{1}{p_{T}^{Anch}(s)}; \frac{\alpha}{\psi}\} \]

\[ C_N(s) = \frac{1 - \alpha}{\psi} \frac{p_{T}^{Anch}(s)}{p_N(s)} = \frac{1 - \alpha}{\psi} \theta(s) \frac{p_{T}^{Anch}(s)}{\mathbb{E}\left[ \frac{p_{T}^{Anch}(s)}{A(s)} \right]} \]

Employment is then given by

\[ L(s) = C_T(s) + C_N(s)/A(s) = \frac{\alpha}{\psi} + \frac{1 - \alpha}{\psi} \theta(s) \frac{p_{T}^{Anch}(s)}{A(s)} \left( \mathbb{E}\left[ \frac{p_{T}^{Anch}(s)}{A(s)} \right] \right)^{-1} \]

It fluctuates with productivity and the actual realization of the traded good price. For the anchor country it only fluctuates together with the markups.

Inflation rates are a function money growth rate, that is determined by the anchor as in a float under discretion

\[ \pi_N(s^t, s_{1t+1}) = \frac{P_N(s^t, s_{1t+1})}{P_N(s^{t-1}, s_{1t})} = \theta(s_{1t}) \max\{A(s_{2t})\} \frac{M_{Anch}(s^t)}{M_{Anch}(s^{t-1})} \]

\[ \pi_T(s^{t+1}) = \frac{P_T(s^{t+1})}{P_T(s^t)} = \frac{A_{Anch}^{s^{t+1}}(s_{1t+1})P_N(s^{t+1})}{A_{Anch}(s^t)\theta(s^t)P_N(s^t)} = \frac{A_{Anch}^{s^{t+1}}(s_{1t+1})}{A_{Anch}(s^t)\theta(s^t)} \max\{A_{Anch}(s^{t+1})\} \frac{M_{Anch}(s^t)}{M_{Anch}(s^{t-1})} \]

Inflation of traded goods is only a function of anchor variables, while inflation of non-traded goods in the client country depends on anchor and client variables.
**Peg to a Discretionary Anchor** Under discretion with a peg prices of traded goods are as prices under discretion for the anchor, for non-traded good prices domestic markups and productivity of the anchor are decisive.

\[
p_T = \frac{\psi}{\alpha - (1 - \alpha)(1 - \theta^{Anch})}
\]

\[
p_N(s') = \frac{1}{\theta(s_{1t})} \frac{1}{A^{Anch}(s_{2t})} \frac{\psi}{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s_{1t}))}
\]

Money growth rate is the one of the anchor and given by

\[
\frac{M(s')}{M(s^{t-1})} = \beta \frac{\alpha}{\psi} \frac{\psi}{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s'))}
\]

Inflation in the client country is then given by

\[
\pi_T(s') = \frac{P_T(s^{t+1})}{P_T(s^t)} = \frac{M^{Anch}(s')}{M^{Anch}(s^{t-1})} \frac{A^{Anch}(s') \theta(s')}{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s_{1t}))}
\]

\[
\pi_N(s') = \frac{P_N(s^{t+1})}{P_N(s^t)} = \frac{M^{Anch}(s')}{M^{Anch}(s^{t-1})} \frac{A^{Anch}(s') \theta(s')}{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s_{1t}))}
\]

while inflation of non-traded goods in the anchor country is

\[
\pi^{Anch}_N(s') = \frac{P_N(s^{t+1})}{P_N(s^t)} = \frac{M^{Anch}(s')}{M^{Anch}(s^{t-1})} \frac{A^{Anch}(s') \theta^{Anch}(s')}{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s_{1t}))}
\]

Note that inflation of non-traded good is different. For both countries their own corresponding markup shocks play a role. As the correlation of markups between countries is not perfect, but rather zero in the iid example here, this implies that volatility of non-traded goods for the client country is lower than for the anchor country. This is because if \(\frac{\theta^{Anch}(s')}{\theta^{Anch}(s_{1t})}\) is large \(\frac{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s_{1t}))}{\alpha - (1 - \alpha)(1 - \theta^{Anch}(s_{1t}))}\) is large as well. Overall \(\pi^{Anch}_N\) is more volatile than \(\pi_N\) if the underlying markup shock process is the same and uncorrelated to the process in the anchor country.

**Discretion and Currency Union** There is a mass of \(n^N\) northern countries and \(n^S\) southern countries. The relative weight of north is then \(\lambda^N = \frac{n^N}{n^N + n^S}\). The union-wide central bank chooses a traded good price for the union taking the non-traded good prices as given. The
current state is \( s = (z, p^l_N(z, v)) \), the primal problem is then

\[
\begin{align*}
\max_{p_T} & \quad \sum_{i \in (N, S)} \lambda^i \sum_v g(v^i) \left[ \alpha \log C^i_T(s^i) + (1 - \alpha) \log C^i_N(s^i) - \psi \left( L^i(s^i) \right) \right] \\
\text{s.t.} & \quad L^i(s^i) = \frac{C^i_N(s^i)}{\lambda^i(s^i)} + C^i_L(s^i) \\
& \quad C^i_T(s^i) = \frac{1}{p_T} \\
& \quad C^i_N(s^i) = \frac{1 - \alpha}{\psi} \frac{p_T(s^i)}{p^i_N(s^i-1, s_{11})} \\
& \quad U_T(s^i) = \frac{U_T(s^i)}{U^i_N(s^i)}
\end{align*}
\]

where \( g(v) \) is again just the average of the union, given the aggregate state. The first order condition is given by:

\[
0 = (1 - 2\alpha) \frac{1}{p_T} + \psi \frac{1}{p_T^2} - (1 - \alpha) \sum_{i = N, S} \lambda^i \sum_v g(v) \frac{1}{p^i_N(z, v)A^i}
\]

We can solve the quadratic equation to get the monetary authorities best response:

\[
p_T \left( z, \{ p^i_N(z_1, v_1) \} \right) = \frac{(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4 \sum_{i = N, S} \lambda^i \sum_v g(v) \left( \frac{1 - \alpha}{\lambda^i(s^i, v_2)} \frac{1}{p^i_N(z_1, v_1)} \right) \psi}}{\sum_{i = N, S} \lambda^i \sum_v g(v) \frac{2(1 - \alpha)}{\lambda^i(s^i, v_2)} \frac{1}{p^i_N(z_1, v_1)}},
\]

As a next step consider again the pricing equation of firms in country \( i \): \( p^i_T = \mathbb{E}_v \left( \frac{1}{A^j} \right) \frac{1}{\theta^j} p^j_T \). As with a single country under discretion, we can solve the problem by plugging in the reaction functions into each other, this gives a fixed point problem and can explicitly be solved for \( p_T \). Let \( \sum_v g(v) \frac{1}{\lambda^i(s^i, v_2)} p^i_N(z_1, v) = \mathbb{E}_v \left[ \frac{1}{\lambda^i(s^i, v_2)} \right] \). Then

\[
p^i_N = \mathbb{E}_v \left( \frac{1}{A^j} \right) \frac{1}{\theta^j} \left( 1 - \alpha \right) p^j_T \sum_{i = N, S} \lambda^i \mathbb{E}_v \left[ \frac{1}{\lambda^i(s^i, v_2)} \right] \theta^j A^j - (1 - 2\alpha)
\]

For the \( p^j_N \) on the right hand side of the equation, plug in \( p^j_N = \mathbb{E}_v \left( \frac{1}{A^j} \right) \frac{1}{\theta^j} p^j_T \)

\[
p^j_N = \mathbb{E}_v \left( \frac{1}{A^j} \right) \frac{1}{\theta^j} \left( 1 - \alpha \right) \sum_{i = N, S} \lambda^i \mathbb{E}_v \left[ \theta^j \| z \right] \theta^j A^j - (1 - 2\alpha)
\]

This gives \( p_T \)

\[
p_T = \frac{\psi}{\left( 1 - \alpha \right) \sum_{i = N, S} \lambda^i \mathbb{E}_v \left[ \theta^j \| z \right] - (1 - 2\alpha)}
\]
Compared to the peg function of aggregate union-wide shocks. This gives rise to the “Union constraint” from markup variations over time (as money growth rate becomes less erratic as well). This term would be constant (1) over time. Volatility of inflation would then only originate markups in the union, and not out of markups of the anchor only. If the union is really large, and the prices of non-traded goods to maximize an equally weighted average of all countries.

\[
C_T = \frac{1}{p_T} = \frac{\alpha}{\psi} - \frac{1 - \alpha}{\psi} \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left(\theta^i \mid z\right)\right)
\]

and \(C_N\)

\[
C_N^i = \frac{1 - \alpha}{\psi} \mathbb{E}_v \left(\frac{1}{A^i}\right)^{-1} \theta^i(s)
\]

Money growth rate is

\[
\Delta M = \beta \frac{\alpha}{\psi} p_T = \beta \frac{\alpha}{\psi} \left(1 - \alpha\right) \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right] - (1 - 2\alpha)
\]

Inflation for the (former) client are then given by

\[
\pi_T(s^t) = \frac{P_T(s^{t+1})}{P_T(s^t)} = \frac{M(s^t)}{M(s^{t-1})} \frac{\alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}{
\alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}
\]

\[
\pi_N(s^t) = \frac{P_N(s^{t+1})}{P_N(s^t)} = \frac{M(s^t)}{M(s^{t-1})} \frac{\mathbb{E}_v \left(\frac{1}{A^i(s^t)}\right)^{-1} \theta(s^t) \alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}{
\mathbb{E}_v \left(\frac{1}{A^i(s^{t+1})}\right)^{-1} \theta(s^{t+1}) \alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}
\]

For the former anchor country, inflation of non-traded goods is

\[
\pi_N(s^t) = \frac{P_N(s^{t+1})}{P_N(s^t)} = \frac{M(s^t)}{M(s^{t-1})} \frac{\mathbb{E}_v \left(\frac{1}{A^i(s^t)}\right)^{-1} \theta^{Anch}(s^t) \alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}{
\mathbb{E}_v \left(\frac{1}{A^i(s^{t+1})}\right)^{-1} \theta^{Anch}(s^{t+1}) \alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}
\]

Compared to the peg \(\frac{\alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}{\alpha - (1 - \alpha) \left(1 - \sum_{i=1}^{N,S} \lambda^i \mathbb{E}_v \left[\theta^i \mid z\right]\right)}\) consists now out of the weighted average of markups in the union, and not out of markups of the anchor only. If the union is really large, this term would be constant (1) over time. Volatility of inflation would then only originate from markup variations over time (as money growth rate becomes less erratic as well).

**Commitment and Currency Union** In a monetary union, the exchange rate is fixed and set to \(e(s^t) = 1\) for all states. This implies that \(P_T\) cannot vary across countries and is only a function of aggregate union-wide shocks. This gives rise to the “Union constraint”

\[
\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\]

The central bank acts under commitment and chooses the union-wide price of traded goods and the prices of non-traded goods to maximize an equally weighted average of all countries.

\[
\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\]

\[
\frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\]
of the world:

$$\max_{P_T, P_N(v)} \mathbb{E}_0 \left[ \sum_{t=1}^{\infty} \sum_{v^T} \beta^t g(v^T) \left[ \alpha \log C_T(z^T, v^T) + (1 - \alpha) \log C_N(z^T, v^T) - \psi \left( L(z^T, v^T) \right) \right] \right]$$

s.t.  

$$L(s^t) = \frac{C_N(s^t)}{A(s^t)} + C_L(s^t)$$

$$C_T(s^t) = \frac{\alpha}{\psi}$$

$$C_N(s^t) = \frac{1 - \alpha}{\psi} \frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})}$$

$$\sum_{s_{2t}} h \left( s^t \mid s^{t-1}, s_{1t} \right) C_T \left( s^t \right) \left[ U_N \left( s^t \right) + \frac{1}{\theta(s_{1t})} \frac{U_L \left( s^t \right)}{A(s^t)} \right] = 0$$

$$\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}$$

where $$\sum_{v^T} g(v^T)$$ simply sums up all the countries. Remember that $$s^t = (z^t, v^t)$$ where $$z^t$$ is the aggregate shock and $$v^t$$ is the country-specific shock. Optimally, the cash in advance constraint does not bind to avoid losses in consumption. The central bank sets prices such that it stabilizes the average of the whole union:

$$\frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})} = \theta(s_{1t}) \left( \sum_{v_{2t}} g(v_{2t}) \frac{1}{A(z_{2t}, v_{2t})} \right)^{-1}$$

As the exchange rate is fixed, prices of traded goods are the same for all countries and the only thing the union-wide central bank can do is to set relative prices equal to the markup times the average productivity of the union.

Consumption and labor are then given by

$$C_T(s^t) = \frac{\alpha}{\psi}, \quad C_N(s^t) = \frac{1 - \alpha}{\psi} \frac{\theta(s_{1t})}{E_t \left( 1/A(v_{2t} \mid z_{2t}) \right)}$$

$$L(s^t) = \frac{C_N(s^t)}{A(s^t)} + C_L(s^t)$$

Consumption of traded goods is as with a flexible exchange rate under commitment (Section 2.3.1) as the cash in advance constraint is not binding. The difference is that consumption of non-traded goods now depends on average productivity in the currency union, as the central bank now conditions its policy on the average of the union and not on each individual country. This will in general be costly for the economy, as the central bank is not able to eliminate all costs coming from rigid prices. For some countries, monetary policy will be too expansionary, for some it will be too contractionary.

Nominal prices, interest rates and money growth rates are computed by resolving the indeterminacy problem in the same way as before. Let $$X(z_{2t}) = \sum_{v_{2t}} g(v_{2t}) \frac{1}{A(z_{2t}, v_{2t})}$$. Consider the lowest possible value of X. That corresponds to the highest possible aggregate productivity value and assume that the cash in advance constraint is exactly binding in this state. Prices
If productivity is not stochastic, then money growth is simply $\beta < 1$.

In the model of Chari et al. (2020a), cooperative and non-cooperative equilibria coincide.

The nominal interest rate in the currency union is given by the Euler equation as before:

$$R(s^t) = \left( \sum_{s^{t+1}} h(s^{t+1} | s^t) \frac{\min_{z_2} \{X(z_{2t})\}}{X(z_{2t+1})} M(s^{t-1}) \right)^{-1}$$

Inflation rates are:

$$\pi_N(s^t, s_{1t+1}) = \frac{P_N(s^t, s_{1t+1})}{P_N(s^{t-1}, s_{1t})} = \theta(s_{1t+1}) \frac{\psi \min_{z_2} \{X(z_{2t+1})\}}{\min_{z_2} \{X(z_{2t+1})\}} \frac{s_2 \theta(s_{1t})}{\min_{z_2} \{X(z_{2t})\}} \frac{\frac{1}{s_2} \frac{1}{A(s_2)} \frac{1}{X(z_{2t})}}{M(s^{t-1})} M(s^t)$$

If productivity is not stochastic, then money growth is simply $\beta < 1$. For inflation this means $\pi_N = \Delta \theta \beta$, $\pi_T = \beta$. Nominal interest rates are then $R = 1$. As in the case with monetary policy under commitment with flexible exchange rates, the union-wide central bank follows the Friedman rule as well. This implies a continued contraction in money supply, zero interest rates and deflation.

Main text from before

\[ \text{B.4.1 Flexible Exchange Rates: Monetary Policy under Commitment} \]

The central bank conducts monetary policy under commitment. This means that the central bank maximizes ex ante lifetime utility of its representative household. It chooses an appropriate state-contingent path of prices subject to the consumer and firm first order conditions, the resource constraint, as well as the production function \(^{14}\). The central bank sets its policy

\(^{14}\)The central banks could also jointly maximize a weighted sum of all countries using their policy instrument for each country. As there are no externalities in the model of Chari et al. (2020a), cooperative and non-cooperative equilibria coincide.
The interpretation of that policy rule is straightforward: After productivity has realized the central bank makes sure that relative prices move in such a way that they replicate the outcome as if non-traded good prices were flexible. This way the central bank can eliminate any distortions coming from rigid prices. The central bank engineers a movement of the exchange rate in such a way that relative prices align. For example, if productivity of the non-traded goods sector is high today, $P_N$ should decrease as it is easier to produce that good. As prices of that good do not adjust, the central bank instead uses the exchange rate to let the currency depreciate so such $P_T$ rises, which means that the relative price for $P_N$ falls. The movement of the exchange rate aims to replicate the outcome of relative prices as if all prices were flexible.

Note also, that optimal monetary policy would never cause consumers to lose consumption because they do not have enough cash. Therefore, the cash in advance constraint is never binding in a way that would lower the household’s consumption. That is the reason why the consumer first order condition with respect to $C_T$ has a multiplier from the cash in advance constraint equal to zero.
B.4.2 Flexible Exchange Rates: Monetary Policy under Discretion

Now consider how a non-credible central bank sets monetary policy. The important difference when a central bank acts under discretion is that it takes the price of non-traded goods as given, as firms have set their prices before the central bank acts. As a consequence, the central bank will try to use monetary policy to inflate away the inefficient monopolistic markups and implement an allocation, that equalizes household’s marginal rate of substitution with the marginal rate of transformation of the economy. That is \( P_T(s^t)/P_N(s^{t-1}, s_{1t}) = A(s_{2t}) \).

In order to do that the central bank will go so far to make the cash in advance constraint binding. As long as this constraint is slack, the central bank can use more inflation to reduce the markups. Therefore, the central bank makes the cash in advance constraint binding and ultimately trades off the costs of markups with the costs of surprise inflation that lower the household’s purchasing power. A central bank under discretion therefore chooses \( p_T(s^t) = P_T(s^t)/M(s^{t-1}) \) to maximize the following problem:

\[
\max_{p_T(s^t)} \mathbb{E}_t \left[ \sum_{\tau=t} \beta^\tau \left( \alpha \log(C_T(s^\tau)) + (1 - \alpha) \log(C_N(s^\tau)) - \psi L(s^\tau) \right) \right]
\]

s.t. \( C_T(s^t) = \frac{1}{p_T(s^t)}, \)

\( C_N(s^t) = \frac{1 - \alpha}{\psi} \frac{p_T(s^t)}{p_N(s^{t-1}, s_{1t})}, \)

\( L(s^t) = C_T(s^t) + \frac{C_N(s^t)}{A(s^t)}. \)

Note the following differences to the problem before: The central bank’s objective function has an expectation operator that starts in \( t \) as the central bank acts under discretion and does not commit beforehand. The consumption constraint for traded goods is also altered, as the cash in advance constraint is binding. In addition, the central bank does not take the firm’s first order condition into account as it acts under discretion. Chari et al. (2020a) show that the dynamic dimension of this problem does not play a role, so the central bank simply acts as maximizing the per period utility of its household. The best response of the monetary authority is to set the price of traded goods as:

\[
p_T(s_t) = p_N(s_{1t}) A(s_{2t}) \left[ \frac{1}{2(1 - \alpha)} \left( 1 - 2\alpha + \sqrt{1 - 2\alpha^2 + 4(1 - \alpha)} \right) \frac{1}{A(s_{2t}) p_N(s_{1t})} \psi F\left( \frac{1}{p_T(s_t)} p_N(s_{1t}) \right) \right]
\]

where the first part on the right-hand side \( p_N(s_{1t}) A(s_{2t}) \) captures the willingness of the central bank to put the marginal rate of transformation equal to the marginal rate of substitution and \( F(\cdot) \) captures the costs from surprise inflation. If \( p_N \) increases by one, \( p_T \) increases less than one-to-one. In the following we assume as in Chari et al. (2020a) that \( \frac{1}{\sigma(s)} < \frac{1 - \alpha}{1 - 2\alpha} \) so that there is a point where marginal costs of surprise inflation equal their marginal benefits.
This simply bounds markups from above, meaning that it is not possible that reducing markup distortions always exceed the costs of reducing trade goods consumption.

Another aspect that needs to be mentioned is, when productivity is stochastic and is sufficiently low compared to its average value, it can happen that the cash in advance constraint is not binding despite the central bank’s policy. That is if $A p_N < C_T$ then $p_T = p_N A$. Taken this into account as well, it implies that the price of traded goods is described by

$$p_T (s_t) = \max \{ p_N (s_{1t}) A (s_{2t}), p_N (s_{1t}) A (s_{2t}) F (\cdot) \}.$$ 

For policy under discretion, it is also important to consider the firms. They take into account that the central bank will try to inflate away their markups. Optimally firms still set prices of traded goods as in (1). Remember that firms observe the markup shock and then set their price taking their expectation for future productivity into account. Overall, the price of traded goods in the equilibrium solves the fixed-point problem of equaling the optimal price firms would set and what the central bank wants to implement. So, in equilibrium, any attempt of the central bank to inflate away the markup is frustrated, as firms anticipate the central bank’s move and set their prices accordingly. The only thing the central bank achieves is an inflationary bias with higher volatility of prices and consumption.

### B.4.3 Unilateral Peg to a Stable Anchor

Consider now the case in which one country (the client country) pegs its currency to a stable country (the anchor). The anchor is assumed to conduct monetary policy under commitment, as in Section 2.3.1. The client country then ensures that the exchange rate to the anchor country stays constant at all points in time. This implies that monetary policy of the client loses its independence and follows the anchor. The main difference to this regime and a currency union is that the client country has no influence how the anchor conducts monetary policy.

In a currency union the union-wide central bank considers all its member states. The central bank of the anchor country then sets relative prices like

$$\frac{P_T (s_t)}{P_N (s_t^{t-1}, s_{1t})} = \theta (s_{1t}) A (s_{2t}).$$

The peg implies that the price of traded goods is the same for both countries. Firms of the client country realize that monetary policy will be as in the anchor country. After markup shocks have realized in the anchor country, they form expectations about productivity and how the central bank of the anchor chooses the price of traded goods. In general, distortions coming from productivity fluctuations will be completely offset in the anchor country, while they will be present in the client country. These distortions are reflected in a volatile movement of employment. There is no inflationary bias in any of both countries.

### B.4.4 Currency Union: Monetary Policy under Commitment

In a monetary union, the exchange rate is fixed and set to $e (s_t) = 1$ for all states. This implies that $P_T$ cannot vary across countries and is only a function of aggregate union-wide shocks.
This gives rise to the “Union constraint”

\[
\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\]

The union consists out of many blocks, each block \(i\) having a mass of countries \(n_i\). The relative weight of block \(i\) is \(\lambda^i = \frac{n_i}{\sum n_i}\). Countries are all the same across blocks, except for the shock process of their markup. The central bank acts under commitment and chooses the union-wide price of traded goods and the prices of non-traded goods to maximize an equally weighted average of all countries of the world:

\[
\max_{P_T, P_N(v)} \mathbb{E}_0 \left[ \sum \lambda^i \sum v^i \left[ \alpha \log C_T^i(s^t) + (1 - \alpha) \log C_N^i(s^t) - \psi (L^i(s^t)) \right] \right]
\]

s.t. \(L(s^t) = \frac{C_N(s^t)}{A(s_{2t})} + C_L(s^t)\)

\(C_T(s^t) = \frac{\alpha}{\psi}\)

\(C_N(s^t) = \frac{1 - \alpha}{\psi} \frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})}\)

\[
\sum s_{2t} h(s^t | s^{t-1}, s_{1t}) C_N(s^t) \left[ U_N(s^t) + \frac{1}{\theta(s_{1t})} \frac{U_L(s^t)}{A(s_{1t})} \right] = 0
\]

\[
\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\]

where \(\sum v^i g(v^i)\) simply sums up all the countries within a block. Remember that \(s^t = (z^t, v^t)\) where \(z^t\) is the aggregate shock and \(v^t\) is the country-specific shock. Optimally, the cash in advance constraint does not bind to avoid losses in consumption. The central bank sets prices such that it stabilizes the average of the whole union:

\[
\frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})} = \theta(s_{1t}) \left( \sum v_{2t} g(v_{2t}) \frac{1}{A(z_{2t}, v_{2t})} \right)^{-1}
\]

As the exchange rate is fixed, prices of traded goods are the same for all countries and the only thing the union-wide central bank can do is to set relative prices equal to the markup times the average productivity of the union.

### B.4.5 Currency Union: Monetary Policy under Discretion

In a monetary union, the exchange rate is fixed and set to \(e(s^t) = 1\) for all states. This implies that \(P_T\) cannot vary across countries and is only a function of aggregate union-wide shocks. This gives rise to the “Union constraint”

\[
\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\]
The union consists out of many blocks, each block \( i \) having a mass of countries \( n_i \). The relative weight of block \( i \) is \( \lambda_i = \frac{n_i}{\sum n_i} \). Countries are all the same across blocks, except for the shock process of their markup. The central bank acts under discretion and chooses the union-wide price of traded goods to maximize an equally weighted average of all countries of the world. The union-wide central bank chooses a traded good price for the union taking the non-traded good prices as given.

\[
\max_{p_T} \sum_i \lambda_i \sum_v g(v) \left[ \alpha \log C_T^i(s^t) + (1 - \alpha) \log C_N^i(s^t) - \psi \left( L^i(s^t) \right) \right]
\]

s.t. \( L^i(s^t) = \frac{C_N^i(s^t)}{A^i(s_{2t})} + C_L^i(s^t), \)
\( C_T^i(s^t) = \frac{1}{p_T(s^t)}, \)
\( C_N^i(s^t) = \frac{1 - \alpha}{\psi} \frac{p_T(s^t)}{p_T^i(s^{t-1}, s_{1t})}, \)
\( \frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(s^t)}{U_N(s^t)}, \)

where \( g(v) \) gives the average state of all countries within a block, given the aggregate state. The policy of the central bank implies to set the price of traded goods such that:

\[
p_T \left( z, \left\{ p_N^i \left( z_1, v_1 \right) \right\} \right) = \frac{1 - 2\alpha}{\sum_{i=N,S} \lambda_i \sum_v g(v)} + \sqrt{(1 - 2\alpha)^2 + 4 \sum_{i=N,S} \lambda_i \sum_v g(v) \frac{(1 - \alpha)}{A^i(z_{2t}, v_2)} \frac{1}{p_N^i(z_1, v_1)}}.
\]

Compared to the policy rule under discretion with an independent national central bank single country-specific shocks are replaced by the average shock realization of the union. As before, firms anticipate the policy of the central bank and take this into account when setting their prices. In a currency union however, they realize that the central bank will only react to the average temptation shock, not the country-specific one. The result is still more inflation. The next section discusses how the policy under discretion in a currency union can still yield some benefits compared to discretion of a single country.

**B.4.6 Degrees of Credibility**

We follow Schaumburg and Tambalotti (2007) and introduce credibility into the model, by assuming that the central bank acts under discretion in a period with a certain probability. The interpretation is that a new governor gets selected with probability \( \xi \) in every period. If a new governor is selected, she acts under discretion in the first period and commits to policy thereafter as long as she is in office. It is not possible to restrain the successor. Formally, there is a sequence of Bernoulli signals \( \eta_t \), with probability \( \xi \) \( \eta_t \) is one and a new governor is chosen, otherwise \( \eta_t \) is zero and the old governor stays in place. We assume that this signal is known before productivity has realized. This implies that firms know if monetary policy acts under commitment or under discretion in a certain period. The optimization problem of the central
Theorem 1

Theorem 2

B.5 Proofs of Theorems

The central bank ends up with a policy rule that is either discretionary or commitment based:

\[
p_T = \begin{cases} 
    p_N(s_{1t}) A(s_{2t}) \theta(s_{1t}) & \text{if } \eta_t = 0 \\
    p_N(s_{1t}) A(s_{2t}) \frac{1}{2(1-\alpha)} \left[ (1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha) \frac{1}{A(s_{2t})} \psi} \right] & \text{if } \eta_t = 1 
\end{cases}
\]

Firms set their prices accordingly to each regime. Overall, average inflation over a longer time horizon for a country is then the weighted average of inflation under discretion and under commitment. The weights correspond to the credibility parameter \( \xi \) that determines the probability of having commitment and discretion. If a country in this setup decides to peg its currency to a stable anchor, the probability of being in a discretionary regime decreases to the level of the anchor country. In a currency union, the central bank is as credible as the most credible member state.

Theorem 1

Inflation falls if a less credible country pegs to more credible anchor. Proof: \((1 - \xi) \frac{\theta(s)}{\theta(s')} + \xi \frac{\theta(s)}{\theta(s')} \beta \frac{\alpha}{1-\alpha} (1-\theta(s)) \Theta(s') > (1-\xi^{\text{Anch}}) \frac{\theta(s)}{\theta(s')} \beta \frac{\alpha}{1-\alpha} (1-\theta(s^{\text{Anch}})) \Theta^{\text{Anch}}(s')\) Assuming that the underlying shock process of \( \theta \) is the same for both countries, the difference between \( \xi \) and \( \xi^{\text{Anch}} \) is decisive for the average inflation rate. \(-\xi + \xi^{\text{Anch}} \frac{\alpha}{1-\alpha} (1-\theta(s)) \Theta(s') > -\xi^{\text{Anch}} + \xi^{\text{Anch}} \frac{\alpha}{1-\alpha} (1-\theta^{\text{Anch}}(s)) \Theta^{\text{Anch}}(s') \Rightarrow \xi > \xi^{\text{Anch}}\) (note that \(\frac{\alpha}{1-\alpha} (1-\theta(s)) \Theta(s') - 1 > 1\). As the markup shock process is the same, the expected value of the right hand side is 1. Then we have \( \xi > \xi^{\text{Anch}} \) which is true as the anchor is more credible and less likely to act under discretion. For the currency union the same is true as the most credible anchor determines monetary policy.

Theorem 2

Inflation volatility. Proof: Compare inflation for the anchor country in a pegged regime and in a currency union regime and assume that the anchor is also the most credible
country. \( \pi_{Anch,Peg}^N = (1 - \xi_{Anch}) \frac{\theta_{Anch}(s)}{\theta_{Anch}(s')} \beta + \xi_{Anch} \frac{\theta_{Anch}(s)}{\theta_{Anch}(s')} \beta \frac{\alpha - (1 - \alpha)}{1 - \beta_{Anch}(s)} \Theta_{Anch}(s') \)

\( \pi_{Anch,Union}^N = (1 - \xi_{Anch}) \frac{\theta_{Anch}(s)}{\theta_{Anch}(s')} \beta + \xi_{Anch} \frac{\theta_{Anch}(s)}{\theta_{Anch}(s')} \alpha - (1 - \alpha)(1 - \beta_{Anch}(s)) \Theta^U(s') \)

with \( \Theta(s') = \frac{\alpha - (1 - \alpha)}{\beta_{Anch}(s')} \) and \( \Theta^U(s') = \frac{\alpha - (1 - \alpha)(1 - \beta_{Anch}(s'))}{\beta_{Anch}(s')} \)

The first component under commitment is the same, while only the second under discretion is different. As the main text mentioned, \( \Theta^U \) is less volatile than \( \Theta_{Anch} \) as the average change of markups between countries is less volatile than the change of markups of one country. In addition to that, the correlation between \( \frac{\theta_{Anch}(s)}{\theta_{Anch}(s')} \) and \( \frac{\alpha - (1 - \alpha)(1 - \beta_{Anch}(s'))}{\beta_{Anch}(s')} \) is lower in a union than this object in a peg: \( \frac{\theta_{Anch}(s)}{\theta_{Anch}(s')} \) and \( \frac{\alpha - (1 - \alpha)(1 - \beta_{Anch}(s'))}{\beta_{Anch}(s')} \).

This leads to a reduction of volatility of inflation for the anchor country. For a client country, the opposite would be true, as its markup realization now play a role for a discretionary monetary authority. Then, only \( \Theta^U \) lowers the volatility while a potentially higher correlation increases volatility for a client country entering the union.

**Theorem 3** Output of N in a peg is higher than in a float if \( \xi > \xi_{Anch} \), proof: \( (1 - \xi) \frac{\alpha}{\psi} + \xi \left( \frac{\alpha}{\psi} \right) < (1 - \xi_{Anch}) \frac{\alpha}{\psi} + \xi_{Anch} \left( \frac{\alpha}{\psi} \right) \Rightarrow (\xi_{Anch} - \xi) \frac{\alpha}{\psi} < (\xi_{Anch} - \xi) \left( \frac{\alpha}{\psi} \right) \). As \( \xi_{Anch} < \xi \), we arrive at \( \frac{\alpha}{\psi} > (1 - \alpha)(1 - \theta) \) which is true for \( \alpha \) and \( \theta \in (0, 1) \). The same is true for all countries entering the currency union, whose credibility is lower than the most credible country.

### B.6 Computation of Interest Rates

**Flexible exchange rate and commitment**

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') U_N(s^{t+1}, s_{1t+1}) P_N(s^{t-1}, s_{1t}) U_N(s^{t})
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{C_N(s^{t'})} \frac{1}{\theta(s_{1t+1})} \frac{1}{\alpha} \frac{1}{\max \{A(s_{2t+1})\}} M(s^{t-1})
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{C_N(s^{t'})} \frac{1}{\theta(s_{1t+1})} \frac{1}{\alpha} \frac{1}{\max \{A(s_{2t+1})\}} M(s^{t-1})
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{\alpha} \theta(s_{1t}) A(s_{2t}) \frac{1}{\theta(s_{1t+1})} \frac{1}{\alpha} \frac{1}{\max \{A(s_{2t+1})\}} M(s^{t-1})
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{\alpha} \frac{1}{\max \{A(s_{2t+1})\}} M(s^{t-1})
\]

\[
\bar{Q}(s') = \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{\alpha} \frac{1}{\max \{A(s_{2t+1})\}} M(s^{t-1})
\]

Interest rates are zero if productivity is not stochastic.
Flexible exchange rates and discretion

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{C_N(s') P_N(s^{t-1}, s_{1t})}{C_N(s^{t+1}) P_N(s^t, s_{1t+1})}
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1-\alpha}{\psi} A(s') \bar{\theta}(s') \frac{p_N(s^{t-1}, s_{1t}) M(s^{t-1})}{\psi} A(s^{t+1}) \bar{\theta}(s^{t+1}) P_N(s^t, s_{1t+1}) M(s^t)
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{\psi} A(s'^{t+1}) \bar{\theta}(s'^{t+1}) \frac{p_N(s^{t-1}, s_{1t}) M(s^{t-1})}{\psi} \beta s A(s_{2t}) \bar{\theta}(s_{1t}) P_N(s^{t-1}, s_{1t})
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1}{\psi} A(s'^{t+1}) \bar{\theta}(s'^{t+1}) \frac{1}{\psi} \frac{1}{\beta \alpha} \sum_{s^{t+1}} h(s^{t+1} | s') \frac{\alpha - (1-\alpha)(1-\theta(s_{1t+1}))}{\psi} < \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{\alpha}{\psi} \frac{\beta \alpha}{\beta \alpha}
\]

\[
R(s^{t+1})^{-1} = t \left[ \frac{\alpha - (1-\alpha)(1-\theta(s_{1t+1}))}{\alpha} \right]
\]

which implies that \(\bar{Q}^{disc}(s') < \bar{Q}^{Commit}(s')\) and therefore \((1+i)^{disc} > (1+i)^{commit}\).

In a currency union with commitment

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{C_N(s') P_N(s^{t-1}, s_{1t})}{C_N(s^{t+1}) P_N(s^t, s_{1t+1})}
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{1-\alpha}{\psi} P_T(s') \frac{p_N(s^{t-1}, s_{1t}) M(s^{t-1})}{\psi} P_T(s^{t+1}) P_N(s^t, s_{1t+1})
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{P_T(s')}{P_T(s^{t+1})}
\]

\[
\bar{Q}(s') = \beta \sum_{s^{t+1}} h(s^{t+1} | s') \frac{\psi \min_{z_{2t}} \{X(z_{2t})\}}{\psi \min_{z_{2t+1}} \{X(z_{2t+1})\}} \frac{\alpha}{\alpha} \frac{X(z_{2t})}{X(z_{2t+1})} M(s^{t-1}) M(s^t)
\]

B.7 Overview of all six Regimes

Table B.1: Output under different monetary regimes.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Regime & \( Y_T \) & \( Y_N \) \\
\hline
CF & \( \alpha \psi \) & \( \frac{1-\alpha}{\psi} \theta(s) A(s) \) \\
DF & \( \alpha \psi - \frac{1-\alpha}{\psi} (1-\theta(s)) \) & \( \frac{1-\alpha}{\psi} \theta(s) A(s) \) \\
CP & \( \alpha \psi \) & \( \frac{1-\alpha}{\psi} \theta(s) T \psi(v) (1/A)^{-1} \) \\
CU & \( \alpha \psi - \frac{1-\alpha}{\psi} (1-\sum_i \lambda^i T \psi(s)) \) & \( \frac{1-\alpha}{\psi} \theta(s) T \psi(v) (1/A)^{-1} \) \\
DU & \( \alpha \psi - \frac{1-\alpha}{\psi} (1-\sum_i \lambda^i T \psi(s)) \) & \( \frac{1-\alpha}{\psi} \theta(s) T \psi(v) (1/A)^{-1} \) \\
\hline
\end{tabular}
\caption{Nominal Rates under different monetary regimes.}
\end{table}

Notes: Output of traded goods \((Y_T)\) and non-traded goods \((Y_N)\) under all possible regimes: Commitment and Float (CF), Discretion and Float (DF), Commitment and Peg (CP), Commitment in a Union (CU), Discretion in a Union (DU).

B.8 Model Graphs and Estimation

B.8.1 SMM

Formally, let \( x \) be the data and \( m(x) \) the moments of the data. The corresponding moments of the model are denoted by \( m(\tilde{x}, \nu) \) where \( \nu \) are the parameters of the model. We simulate the model \( S \) times, such that there are \( S \) simulations of the model data \( \tilde{x} = \{ \tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_S \} \).

The vector of moments in one simulation \( s \) of length \( T \) consists out of three expressions. The standard deviation and the mean of a country’s inflation rate during a discretionary float in simulation \( s \) and the average markup

\[
\text{std}(\pi_s) = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\pi_t - \bar{\pi}_s)^2}, \quad \mu(\pi_s) = \frac{1}{T} \sum_{t=1}^{T} \pi_t, \quad \mu(\theta_s) = \frac{1}{T} \sum_{t=1}^{T} \frac{1}{\bar{\theta}_t}
\]

The estimated model moments from the simulation are

\[
\hat{m}(\tilde{x}, \nu) = \frac{1}{S} \sum_{s=1}^{S} m(\tilde{x}_s | \nu).
\]
The SMM approach estimates the parameter vector $\hat{\upsilon}_{SMM}$ to choose $\upsilon$ in such a way that it minimizes the $L^2$ norm of the sum of squared errors in moments. We define the moment error function as the percent difference in the vector of simulated model moments from the data moments

$$e(\tilde{x}, x \mid x) = \frac{\hat{m}(\tilde{x} \mid \upsilon) - m(x)}{m(x)}.$$ 

The SMM estimator is now the following:

$$\hat{\upsilon}_{SMM} = \upsilon : \min_{\upsilon} e(\tilde{x}, x \mid x)^T W e(\tilde{x}, x \mid x)$$

where $W$ is a weighting matrix, in a first step it is the identity matrix, implying equal weights for all moments.

Figure B.1: $\pi_N$ as a function of the markup in a monetary regime under discretion. The markup is defined as $\frac{1}{\theta}$. High markups correspond to a low elasticity of substitution between intermediate goods, allowing those firms to charge high prices. The dashed blue line corresponds to a trade openness of 35%, the solid red line of 25% and the dashed yellow line of 30%.

Table B.3: SMM for 1960-1999
<table>
<thead>
<tr>
<th>Country</th>
<th>( \theta )</th>
<th>( \tilde{\theta} )</th>
<th>( \mu_\theta ) data</th>
<th>( \mu_\theta ) model</th>
<th>( \sigma_\theta ) data</th>
<th>( \sigma_\theta ) model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>0.9517</td>
<td>0.9928</td>
<td>7.03%</td>
<td>7.04%</td>
<td>0.057</td>
<td>0.057</td>
</tr>
<tr>
<td>Germany</td>
<td>0.976</td>
<td>0.991</td>
<td>2.96%</td>
<td>3.16%</td>
<td>0.019</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Figure B.2: Estimated credibility \( \xi \) over time

Notes: Estimated probability of acting under discretion (\( \xi \)). The higher the value the more likely is discretionary policy in the model. The vertical dashed lines indicate exchange rate events of Italy according to Ilzetzki et al. (2019). Orange indicates a move towards a more floating regime, blue towards a more fixed regime.

Next consider the estimated probability of acting under discretion for each country between 1950 to the end. In general, the SMM approach prefers to give estimates of the probability that are close to 1 or to 0. The reason for that is, that intermediate values would imply that countries jump often between commitment and discretionary regimes, which would imply too large inflation volatility. Nevertheless a tendency can be clearly seen: Under fixed exchange rate regimes, credibility tends to be larger for both countries, it is particularly large in the currency union. The floating episode between 1971 and 1985 is characterized by higher inflation rates and for both countries, the estimate for credibility implies that both central banks acted more under discretion. Germany regained credibility quicker however, after both countries entered a more fixed exchange rate regime again.