Supply of Funds Curve in Models of Monetary Policy

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Abstract

In this paper, I match long-run reactions of a representative agent new Keynesian model in which households' utility function is augmented by a preference for bond holding (RANK-BUF) to a calibrated heterogeneous agent model (HANK). I find that the RANK-BUF model displays similar behavior to the HANK model for shocks that induce changes to the real interest rate but that leave other equilibrium prices mostly unchanged. Changes of equilibrium prices alter the profile of risk households face in the HANK model which is a channel that is absent from the representative agent framework. The RANK-BUF model is hence only a good stand-in for shocks that move along the HANK model supply of funds curve.

Keywords: Incomplete Markets, Bonds in Utility, Nominal Rigidities

JEL-Codes: E12, E21, E32

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1 Introduction

A new class of models in which households face uninsurable idiosyncratic income risk and nominal rigidities (the so-called heterogeneous agent new Keynesian models or HANK models) have been at the frontier of macroeconomic research in recent years. Yet while the potential of modeling rich household heterogeneity may be desirable in some applications, the added complexity of both the economic environment as well as the requirement of numerical solution may present a cumbersome hurdle in cases where the aggregates are of primary interest. In an effort to circumvent some of the downsides of HANK models complexity, Kaplan and Violante (2018) proposed a representative agent model in which the utility function of agents is augmented by a preference for positive bond holdings (the RANK-BUF model). The justification is that including bonds in the utility function directly introduces an additional motive for holding bonds akin to the self-insurance in the case of uninsurable idiosyncratic risk. This self-insurance motive introduces positive relationship between equilibrium quantity of safe assets and real interest rate, as analyzed by Aiyagari (1994), and the RANK-BUF model attempts to capture similar considerations in a reduced-form way.

In this paper I perform numerical analysis of the similarity between dynamic responses in a small scale HANK model, the RANK-BUF model, and the standard RANK model. The idea is to discipline the reactions of the RANK-BUF model to shocks by matching its steady state (or long run) supply of funds curve to the one implied by a desired HANK calibration.

I find that the similarity of responses to aggregate shocks depends significantly on the properties of the shock in question. In particular, all three models display similar behavior following a transitory monetary policy shock. This kind of shock ends up not inducing sufficiently persistent change in bond demand or supply which would allow the assumption of valued bonds to play a bigger role in determining the equilibrium outcomes. One can interpret this result through the prism of permanent income behavior – households attempt to smooth transitory income changes and hence display little reaction in their consumption decisions; the consumption smoothing channel dominates for sufficiently transitory shocks. In so far as the other aspects of the economy are held constant across the models, small desired changes in consumption result in similar overall macroeconomic behavior. This result means that it is important to consider highly persistent shocks, at least in this calibration, if one wishes to investigate the possible implications of the variation in the supply of funds curve between RANK model on the one hand and RANK-BUF and HANK models on the other.

Turning to highly persistent shocks, I first consider persistent shocks to government target debt levels. This shock brings out the main difference between the RANK model with its flat supply of funds curve and and the RANK-BUF and HANK models that have matched upward-sloping curve. The result is that the procedure of matching the supply of funds slopes succeeds in making the RANK-BUF and the HANK models behave very similarly whereas the RANK model shows slight deviations, especially in the flexible price limit. Because price stickiness has redistributive effect, it drives the HANK and RANK-BUF models somewhat apart.

I finally consider a highly persistent TFP shock as a way to model a shock shifting the HANK model supply of funds curve. This channel is by definition absent from the representative agent models because the channel operates through changing the risk that households face. I focus on the conservative case with households not facing idiosyncratic risk in profits. I find that the RANK-BUF is not particularly well-suited as an approximation for this shock and behaves instead more similarly to the standard RANK model.

The present work contributes into a broad literature evaluating the implications of incomplete markets for economic dynamics. Specifically, Nakajima (2005) demonstrates analytically that shocks to the discount factor of the representative agent in a real business cycle model can be derived from microeconomic market incompleteness. Building on this early work, Berger, Bocola, and Dovis (2019) establish more generally that a large class of heterogeneous agent economies can be equivalently represented by economies with complete markets and stochastic shocks to preferences. They provide examples of this equivalence in models with conditions guaranteeing no trade in assets (the so-called zero liquidity economies) and with various borrowing limits. Werning (2015) also establishes similar equivalence result in several model economies.

The existence proof notwithstanding, construction of equivalent economies away from the zero liquidity limit is challenging. In this context, Krueger and Lustig (2010) construct such an equivalence under considerably strict assumptions implying constant interest rate even in an enviroment with aggregate risk. In relation to the present work, their assumptions preclude the analysis of liquidity providing role of government debt (or equivalently its role in relaxing the borrowing constraints) which has been one of the hallmarks of heterogeneous agent literature since the work of Aiyagari and McGrattan (1998).

Turning to numerical simulations, Kaplan and Violante (2018) and Auclert, Rognlie, and Straub (2018) explore the similarity of the RANK-BUF models relative to HANK models away from the zero liquidity limit like the present work. The difference lies in the method of model matching – the two papers match short run impulse responses directly without regard for the implied steady state supply of funds curve. Incidentally, the latter set of authors view RANK-BUF specification less favorably than the former.

There are several recent papers that study the implications of including bonds in the utility function. Michaillat and Saez (2018) study the RANK-BUF model under the assumption of zero liquidity given positive marginal utility of wealth at this point. Hagedorn (2018) relates RANK-BUF model to HANK model where the latter is again limited to the zero liquidity. Mian, Straub, and Sufi (2020) justify the assumption of bonds in the utility function by a bequest motive but work with a calibration implying downward sloping steady state supply of funds curve.

The models in question are often compared to the work by Galí, López-Salido, and Vallés (2007) on a spender-saver kind of new Keynesian models (the TANK model). The justification for TANK models comes from estimated positive reaction of consumption to government spending shocks on the macroeconomic level, a relationship that favors the addition of non-Ricardian (spender) households into the economic model. The RANK-BUF model on the other hand is matched to the microeconomic behavior of agents facing uninsurable idiosyncratic income risk. The preference for bond holding in agents' utility function breaks Ricardian equivalence of government financing while the equivalence is preserved in the TANK framework – this means that persistent changes in the amount of issued government bonds impact the level of interest rate in RANK-BUF.

The specific setup of the models I consider follows the model of Bayer, Lütticke, Pham-Dao, and Tjaden (2019) with the exception that I abstract from the capital accumulation (such that I work with a one asset HANK model) and simplify the household side of the economy by assuming uniform distribution of profits.

The remainder of this paper is organized as follows. Section 2 describes the RANK, RANK-BUF, and HANK models I use while Section 3 provides the description of calibration and the benchmark steady state. Section 4 then focuses on the analysis of fluctuations around the benchmark steady state. Section 5 concludes.

2 Setup of the models

This section introduces the specifications of the three economic environments that I consider – the heterogeneous agent new Keynesian (HANK) model, the representative agent new Keynesian model with bonds in the utility function (RANK-BUF model), and the representative agent new Keynesian (RANK) model. Each model consists of the household sector, the government sector, and the firm sector. The latter two sectors are assumed to be identical across the models while the household sector is not – households in the RANK-BUF model have a direct preference for holding bonds in their utility function whereas the HANK model households face uninsurable idiosyncratic income risk. The standard RANK model can hence be understood as a special case of either of the two remaining models.

The government sector that I consider differs from a more standard setup in two ways. Firstly, government levies labor and profit taxes instead of lump-sum taxes. This choice is made because labor and profit taxes apply more naturally in the context of the HANK economy than lump-sum taxes.

Secondly, the fiscal side of the government operates with constant taxation, exogenous bond issuance target, and government spending that adjusts endogenously to balance the budget. I utilize this setting of the fiscal arm in order to avoid the impact of possible changes in taxation which would occur in the regime with active monetary and passive fiscal policy regime if taxation borne the endogenous adjustment and government spending was exogenous. Changes in taxation have direct effects on the level of idiosyncratic risk that households face in the HANK model I consider and therefore can shift the model's supply of funds curve. I focus on the issue of determinacy given various policy regimes in the Appendix A.

I denote the benchmark steady state values by variables without time subscripts throughout the paper. The following subsection describes the complete characterization of the RANK-BUF model while the section thereafter describes the household sector of the HANK model.

2.1 RANK-BUF model

This subsection introduces the RANK-BUF economy as well as the special case of the RANK model which simply abstracts from the assumption of valued bonds.

Households. The economy consists of a unit measure of identical households who face the following problem:

$$\max_{\{c_t, n_t, b_t\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} \left(c_t - \frac{n_t^{1+\varphi}}{1+\varphi} + F(b_t) \right)^{1-\gamma} - \frac{1}{1-\gamma} \right] \tag{1a}$$

$$\text{subject to} \quad c_t + b_t = \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + (1-\tau)(w_t n_t + \Pi_t), \ t \geq 0 \tag{1b}$$

$$b_{-1}, \ i_{-1} \text{ given.} \tag{1c}$$

Households make decisions in period t about real consumption c_t, labor supply n_t, as well as about the amount of real bond holdings b_t carried over to period $t+1$. The inflation level π_t (defined as $1+\pi_t = \frac{P_t}{P_{t-1}}$ in terms of the price level from the firm sector), the real wage level w_t, and real profits Π_t are equilibrium objects with stochastic behavior that households take as given. The nominal interest rate i_t and the amount of real bonds b_t are assumed to be controlled by the government – incorporating standard feedback rules specified below. τ is a time-constant wage and profits tax rate.

The utility function of households takes the standard Greenwood, Hercowitz, and Huffman (1988) (GHH) form augmented with the additional direct preference for holding real bonds, $F(b_t)$. This specification implies that the steady state supply of funds curve of the households is independent of the level of government purchases, a feature that also holds for the version of the HANK model I consider as long as no change is made to the prices households face. I parameterize the function $F(b_t)$ by using the following flexible form:

$$F(b_t) = \xi \frac{(b_t + b^i)^{1-\phi} - 1}{1-\phi},$$

where ξ, b^i, and ϕ are non-negative parameters that can be though of as the slope, intercept, and curvature of the steady state supply of funds curve, respectively. The assumed functional form of F implies that the steady state supply of funds curve can be calibrated as upward-sloping in the (b, r) space, with the upper bound of $\frac{1}{\beta} - 1$. Setting $\xi = 0$ recovers the RANK model specification. $\beta \in (0, 1)$, $\gamma \geq 0$, and $\varphi > 1$ are the remaining parameters.

The first order condition of a household with respect to b_t takes the following form:

$$(-1 + F'(b_t))\,\lambda_t + \beta\,\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\lambda_{t+1}\right] = 0 \tag{2}$$

where

$$\lambda_t = \left(c_t - \frac{n_t^{1+\varphi}}{1+\varphi} + F(b_t)\right)^{-\gamma}. \tag{3}$$

The labor first order condition becomes:

$$(1-\tau)w_t - n_t^{\varphi} = 0. \tag{4}$$

I also define the real interest rate relationship in the following way:

$$1 + r_t = \mathbb{E}_t\,\frac{1+i_t}{1+\pi_{t+1}}. \tag{5}$$

Government. The government in this economy uses tax revenues and sales of new nominal bonds to pay back the old bonds and finance real government purchases g_t. This means that the government budget constraint becomes:

$$g_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} = \tau(w_t n_t + \Pi_t) + b_t. \tag{6}$$

For convenience, I denote the government tax revenues T_t:

$$T_t = \tau(w_t n_t + \Pi_t). \tag{7}$$

I further assume that the nominal interest rate i_t and the issued amount of real bonds b_t evolve according to the following feedback rules:

$$i_t = i + \rho_i(i_{t-1} - i) + \theta_m \pi_t + \varepsilon_{mt}, \tag{8}$$

$$b_t = (1-\rho_b)\bar{b}_t + \rho_b b_{t-1} - \theta_{f\pi}\pi_t - \theta_{fT}(T_t - T), \tag{9}$$

$$\bar{b}_t = (1-\rho_{\bar{b}})b + \rho_{\bar{b}}\bar{b}_{t-1} + \varepsilon_{bt}. \tag{10}$$

Monetary policy follows a standard Taylor rule formulation with interest rate smoothing and is subject to monetary policy shocks. I work with zero inflation level at the benchmark steady state.

Government bonds follow a partial adjustment process represented by equation (8) whereby the level of newly issued bonds b_t shifts in the direction of the current bond target level $\bar{b}_t$. The bond target level itself follows an exogenous process which is subject to stochastic shocks. In steady state, both the actual level of bonds as well as the bond target take on the exogenous value b. Given this bond rule and fixed taxation level, government budget constraint determines the endogenous government spending level, i.e. government spending adjusts endogenously to satisfy the government budget constraint. In addition, government bond issuance reacts to both inflation and tax revenues in order to guarantee determinacy also in case of monetary dominance.

In sum, one can view ρ_i, ρ_b, $\rho_{\bar{b}}$, θ_m, $\theta_{f\pi}$, and θ_{fT} as the parameters defining policies of the government sector. I consider values of ρ_i, ρ_b, and $\rho_{\bar{b}}$ that belong in the interval $[0, 1)$ whereas I leave the sizes of θ_m, $\theta_{f\pi}$, and θ_{fT} open prior to investigating the determinacy of the system.

Firms. The firm sector is composed of competitive final good firms and a unit-mass of monopolistically-competitive intermediate goods firms facing Rotemberg price-setting friction.

Specifically, final good firms combine intermediate goods by using the CES production function:

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

with $\varepsilon > 1$ being the elasticity of intermediate goods substitution and $j \in [0,1]$ indexing the intermediate goods firms. Final good firms hence maximize nominal period profits by choosing quantities of intermediate goods $y_t(j)$ subject to their prices $P_t(j)$:

$$\max_{\{y_t(j)\}_{j=0}^{1}} P_t y_t - \int_0^1 P_t(j) y_t(j) dj.$$

This profit maximization gives the demand curves:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} y_t$$

and the aggregate price index (definition of nominal output):

$$P_t^{1-\varepsilon} = \int_0^1 P_t(j)^{1-\varepsilon} dj.$$

The intermediate goods firms choose their price $P_t(j)$ and the amount of employed labor $n_t(j)$ in order to maximize discounted real period profits:

$$\max_{\{P_t(j), n_t(j)\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{P_t(j)}{P_t} y_t(j) - w_t n_t(j) - \frac{\varepsilon y_t}{2\kappa}\left(\log \frac{P_t(j)}{P_{t-1}(j)}\right)^2\right] \tag{11a}$$

subject to

$$y_t(j) = z_{t-1} n_t(j)^{1-\alpha}, \ t \geq 0 \tag{11b}$$

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} y_t \tag{11c}$$

$$P_{-1}(j), \ z_{-1} \text{ given.} \tag{11d}$$

$\kappa > 0$ is the Rotemberg price adjustment frictions parameter and $1 - \alpha \in (0,1)$ is the labor share in the production function. z_{t-1} is the aggregate total factor productivity following a partial adjustment process:

$$z_t = (1-\rho_z)\bar{z}_t + \rho_z z_{t-1}, \tag{12}$$

$$\bar{z}_t = (1-\rho_{\bar{z}})z + \rho_{\bar{z}} \bar{z}_{t-1} + \varepsilon_{zt}. \tag{13}$$

as for the case of government bonds above. z is a steady state constant chosen to normalize the steady state output to 1.

The presence of the demand equation (11c) in the maximization problem gives rise to a wedge, $mc_t(j)$, in the labor first order condition:

$$w_t = mc_t(j)(1-\alpha) z_{t-1} n_t^{-\alpha}(j)$$

Similarly, the first order condition with respect to the price becomes:

$$(1-\varepsilon)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} \frac{1}{P_t} - \frac{\varepsilon}{\kappa} \log\left(\frac{P_t(j)}{P_{t-1}(j)}\right) \frac{1}{P_t(j)} + \varepsilon mc_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} \frac{1}{P_t(j)} - \\ -\beta\, \mathbb{E}_t \left[\frac{y_{t+1}}{y_t} \frac{\varepsilon}{\kappa} \log\left(\frac{P_{t+1}(j)}{P_t(j)}\right)\left(-\frac{1}{P_t(j)}\right)\right] = 0$$

Absent any heterogeneity in the initial price level $P_{-1}(j)$, the decisions of individual firms need to be identical since they solve identical problems with unique solution given determinacy at the aggregate level. As such, the aggregated conditions become:

$$w_t = mc_t(1-\alpha)z_{t-1}n_t^{-\alpha} \tag{14}$$

$$\kappa\left(\frac{1-\varepsilon}{\varepsilon}+mc_t\right) = \log\left(1+\pi_t\right) - \beta\,\mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\log\left(1+\pi_{t+1}\right)\right] \tag{15}$$

$$y_t = z_{t-1}n_t^{1-\alpha} \tag{16}$$

$$\Pi_t = \left(1-(1-\alpha)mc_t\right)y_t - \frac{\varepsilon y_t}{2\kappa}\left(\log\left(1+\pi_t\right)\right)^2 \tag{17}$$

Equilibrium. Conditions (1b), (2)-(10), (12)-(17) form a dynamic system in b_{t-1}, i_{t-1}, $\bar{b}_{t-1}$, z_{t-1}, $\bar{z}_{t-1}$, c_t, n_t, w_t, g_t, mc_t, π_t, y_t, Π_t, λ_t, r_t, and T. Variables with index $t-1$ represent state variables at time t. In addition, ε_{mt}, ε_{bt}, and ε_{zt} are the exogenous shocks.

Model solution. I linearize the equilibrium conditions around the non-stochastic zero-inflation steady state and solve the resulting linear system with the method of Klein (2000). In the implementation, I include stochastic shocks as state variables.

2.2 The HANK model

This section introduces in greater detail the HANK model. The government and firm sectors are identical to the representative agent model version whereas households now face uninsurable idiosyncratic risk modeled as a Markov chain for labor productivity ξ_{it-1}.

Households. I consider a setting in which a household i faces the following problem:

$$\max_{\{c_{it},n_{it},b_{it}\}_{t=0}^{\infty}} \quad \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{1-\gamma}\left(c_{it}-\xi_{it-1}\frac{n_{it}^{1+\varphi}}{1+\varphi}\right)^{1-\gamma}-\frac{1}{1-\gamma}\right] \tag{18a}$$

$$\text{subject to} \quad c_{it}+b_{it} = \frac{1+i_{t-1}}{1+\pi_t}b_{it-1}+(1-\tau)(\xi_{it-1}w_t n_{it}+\Pi_t),\ \ t\geq 0 \tag{18b}$$

$$b_{it}\geq 0 \tag{18c}$$

$$b_{i,-1},\ i_{-1},\ \xi_{i,-1}\ \text{given.} \tag{18d}$$

The expectation is now taken with respect to both the aggregate shocks as before as well as the development of household idiosyncratic labor productivity. The Markov chain for this idiosyncratic risk is chosen so as to approximate a realistic process for household earnings. I therefore consider the following AR(1) formulation:

$$\log(\xi_{it}) = -\left(\frac{\sigma^2}{2}\right)(1-\rho)+\rho\log(\xi_{it-1})+\varepsilon_t,\ \text{with}\ \varepsilon_t\sim N(0,\sigma^2(1-\rho^2)).$$

Under this formulation, the unconditional mean and variance of ξ_{it} are 1 and $\exp(\sigma^2) - 1$ respectively. The actual Markov chain approximation is then arrived at by using the Tauchen algorithm to create an approximate transition matrix given $\rho \in (0, 1)$ – the parameter governing the persistence of the process. The remaining free parameter σ^2 governs dispersion of the feasible approximating values (i.e. the state space) of ξ_{it-1} around the mean value of 1.

In addition, budget constraint equation (18b) states that firm sector profits are distributed uniformly across households. While there are various economically distinct ways one could model the distribution of profits, I focus on the dispersion in labor income only in the baseline case.

The assumption that the disutility of labor supply is weighted by the current labor productivity implies that labor supply choice of a household is independent of its current idiosyncratic productivity. In the absence of this weighting, labor supply would be higher for more productive agents and lower for less productive agents without affecting the mean value of total labor supply given the unit mean of ξ_{it-1}. The labor first order condition thus becomes:

$$n_{it}^{\varphi} = n_t^{\varphi} = (1 - \tau) w_t$$

which is identical to the representative agent case of equation (4).

The problem in this case includes an explicit borrowing constraint which will typically bind for agents with sufficiently long chains of adverse idiosyncratic shocks. The first order condition of a household i with respect to b_{it} therefore now takes the following form:

$$\left(c_{it} - \xi_{it-1}\frac{n_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} = \beta\,\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\left(c_{it+1} - \xi_{it}\frac{n_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma}\right] \text{ and } b_{it} > 0 \tag{19a}$$

or

$$\left(c_{it} - \xi_{it-1}\frac{n_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \geq \beta\,\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\left(c_{it+1} - \xi_{it}\frac{n_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma}\right] \text{ and } b_{it} = 0. \tag{19b}$$

Equation (18b) as well as condition (19) generalize the equations (1b) and (2) from the representative agent case.

Model solution. In solving the model I use the methodology of Reiter (2009) or Reiter (2010) to incorporate the budget constraint equation (18b) and the Euler condition (19) into the set of equilibrium equations. The relevant state variable becomes the frequency distribution of bond holdings, denoted D_{t-1}.

I approximate the distribution D_{t-1} by considering a fixed set of grid points for the possible levels of bond holdings $b_g = \{0, \ldots, b_{max}\}$. Then the actual approximate distribution assigns a frequency for every element in the Cartesian product of the grid points b_g and the state space of the idiosyncratic labor productivity. Following Reiter (2010), any probability mass of the distribution D_{t-1} prior to the approximation that does not fall on the grid b_g is attributed to the neighboring grid points in a proportional manner. As a result, the budget constraint equation (18b) can be replaced by the Markov chain:

$$D_t = P(i_{t-1}, \pi_t, w_t, n_t, \Pi_t, C_t) \cdot D_{t-1}, \tag{20}$$

where the transition matrix becomes a function of the relevant aggregate and individual level variables. In particular, the expression C_t denotes a vector of consumption function values at the same Cartesian product of the grid points b_g and the idiosyncratic risk state space. Variables D_{t-1} and C_t are defined on the same grid for simplicity.

The control variable C_t needs to be consistent with the Euler condition (19) which I ensure by using the Endogenous Grid Method of Carroll (2006). In the present context it requires using the next period control variables C_{t+1} to calculate (where ξ_{t-1} encodes idiosyncratic risk levels in a way that is conformant with C_t):

$$C_t^{en} = \left(\beta \, \mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}} \left(C_{t+1} - \xi_t \frac{n_{t+1}^{1+\varphi}}{1+\varphi} \right)^{-\gamma} \right] \right)^{-1/\gamma} + \xi_{t-1} \frac{n_t^{1+\varphi}}{1+\varphi}$$

and

$$b_{t-1}^{en} = \frac{1+\pi_t}{1+i_{t-1}} \left(C_t^{en} + b_g - (1-\tau)(\xi_{t-1} w_t n_t - \Pi_t) \right)$$

where *en* stands for the endogenous grid. These values are then interpolated onto the exogenous grid b_g, with the caveat that points falling below the borrowing constraint follow the no-borrowing condition instead.

3 Calibration of models and benchmark steady state

In this section I calibrate the parameters of the models, first the parameters that are joint to both RANK-BUF and HANK models and then the 2 additional parameters of the HANK model. I then use the calibrated HANK model and its steady state supply of funds curve to match the level and the slope of the corresponding curve in the RANK-BUF model. I also describe additional variables of the resulting benchmark steady state, i.e. the steady state in the absence of any shocks.

Table 1 provides the complete calibration of the joint parameters which take mostly standard values. I use $\alpha = 0$ since capital is not considered, $\beta = 0.94$, $\gamma = 2$, and Frisch labor supply elasticity parameter $\varphi = 2$. The price stickiness parameters are also standard, ε of 10 ensures modest steady state level of profits while κ of 0.1 is consistent with prices remaining unchanged for about 4 quarters on average in the equivalent Calvo formulation. The autoregressive TFP parameter ρ_z is calibrated to 0.7.

I consider highly persistent shocks to the target level of government bonds ($\rho_{\bar{b}} = 0.99$) and TFP ($\rho_{\bar{z}} = 0.99$) which allows me to consider, at least to first order approximation, shifts in steady states following a shock to government target bond level. The use of highly persistent shocks is in line with the focus on long-run shifts in government policy while the assumption of linear approximation simplifies the analysis. I calibrate steady state government parameters τ and b to values ensuring reasonable steady state quantity of government spending.

The five parameters of the government sector are calibrated to values used by Bayer et al. (2019). That is, monetary policy feedback parameter θ_m takes the value of 1.25, interest rate smoothing parameter ρ_i equals 0.8, speed of bond adjustment parameter ρ_b equals 0.86, government bonds reaction to inflation parameter $\theta_{f\pi}$ equals 1.5 and to revenues θ_{fT} equals 0.5.

Table 2 contains the additional HANK model parameters. These are the income shock volatility σ, calibrated to the value of 0.6, and the income shock persistence ρ which takes the value 0.98. The persistence value is taken from Bayer et al. (2019) whereas the income shock volatility that I use is approximately double of the value implied by the mean risk level in their calibration. The reason is that my simpler framework abstracts from stochastic volatility so I make the baseline income process more risky.

The values of model variables in benchmark steady state are described in Table 3. The resulting size of equilibrium interest rate is 0.8%.

I also construct a histogram of bond holdings by idiosyncratic risk type (Figure 1). I use 8 idiosyncratic types for the Tauchen approximation of the idiosyncratic risk process which are then bunched by 2 in the histogram. The calibration actually results in a fairly large proportion of households being at their borrowing constraint of 0 (approximately 33%). This large proportion is reassuring given the large number of authors who stress the importance of using models that are realistic along this dimension, among them for example Kaplan and Violante (2014) or McKay, Nakamura, and Steinsson (2016).

I use these parameters together with the joint model parameters to calculate the steady state

Table 1: Parameters joint to both RANK-BUF and HANK models.

Parameter	Value	Description
Parameters affecting benchmark steady state		
α	0	capital share
β	0.94	discounting
γ	2	CRRA
ε	10	intermediate goods elasticity
κ	0.1	Rotemberg price adj. cost
φ	2	labor supply
τ	0.3	steady state taxes
b	0.6	steady state bonds
z	1.17	steady state TFP
Parameters not affecting benchmark steady state		
ρ_z	0.7	persistence of TFP shock
$\rho_{\bar{z}}$	0.99	persistence of TFP target
ρ_i	0.8	interest rate smoothing
ρ_b	0.86	speed of bond adjustment
$\rho_{\bar{b}}$	0.99	persistence of bond target shock
θ_m	1.25	monetary policy inflation response
$\theta_{f\pi}$	1.5	fiscal policy inflation response
θ_{fT}	0.5	fiscal policy revenues response

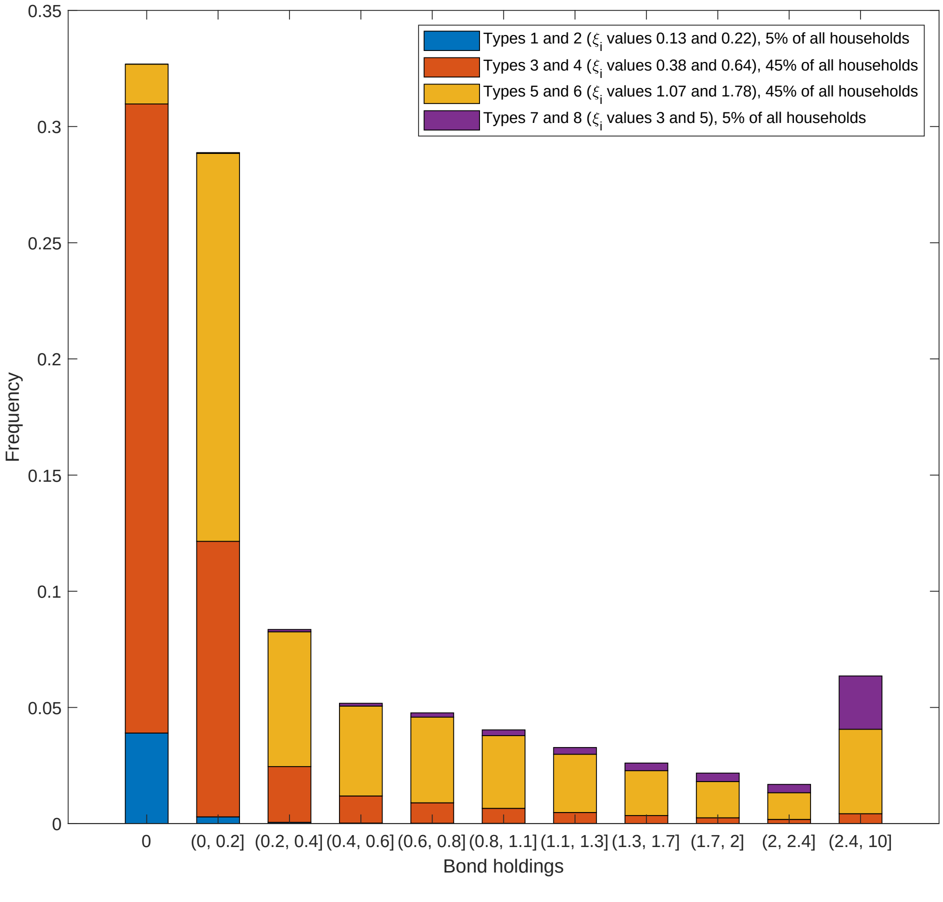
Table 2: HANK model parameters.

Parameter	Value	Description
σ	0.6	income shock volatility
ρ	0.98	income shock persistence

Table 3: Model variables in benchmark steady state.

b, $\bar{b}$	i, r	z	c	n	w	g	mc	π	y	Π
0.6	0.008	1.17	0.71	0.85	1.05	0.29	0.9	0	1	0.1

Figure 1: Distribution of bond holdings at the benchmark steady state



supply of funds curve of the HANK model. This curve is established by starting with an interest rate which implies a consumption function of the household. The consumption function in turn results in a certain steady state quantity (and frequency distribution) of bonds. As higher interest rate makes holding bonds cheaper, households will prefer larger quantity of bonds so as to self-insure against the adverse productivity shocks – it is a well-known result that the bond holdings would actually diverge to positive infinity for gross interest rate approaching the time preference rate $1/\beta$ (Aiyagari, 1994).

The steady state supply of funds curve in the RANK-BUF model can be, in contrast to the HANK model, derived by hand and the GHH assumption guarantees that it will be invariant across steady states. The Euler condition (2) evaluated at steady state demonstrates that it is the marginal utility of wealth holding, $F'(\mathbf{b})$, that affects the steady state supply of funds curve in RANK-BUF model:

$$1 + \mathbf{r} = \frac{1 - F'(\mathbf{b})}{\beta}.$$

I use here $\mathbf{b}$ and $\mathbf{r}$ to represent generic steady state value, instead of the benchmark steady state. The fact that supply of funds curve in RANK-BUF is independent of other equilibrium variables is no different from the standard RANK model where the gross interest rate is fixed at $1/\beta$ in any steady state. Linearization of the Euler equation around the benchmark steady state gives the slope of the supply of funds:

$$\frac{-F''(b)}{1 - F'(b)}(b_t - b) + \frac{\lambda_t - \lambda}{\lambda} = \frac{r_t - r}{1 + r} + \mathbb{E}_t \frac{\lambda_{t+1} - \lambda}{\lambda}.$$

HANK model parameters can therefore determine $F'(b)$ and $F''(b)$ from steady state considerations. The calibration of the HANK model then implies the first derivative of function F at the steady state bond quantity of $F'(b) = 0.05$, the second one of $F''(b) = -0.04$.

The last condition that will end up affecting RANK-BUF model's impulse responses is size of $F(b)$ – this can be seen from the remainder of the linearized Euler equation which reads:

$$\frac{-\gamma}{c - \frac{n^{1+\varphi}}{1+\varphi} + F(b)} \left[(c_t - c) - n^{\varphi}(n_t - n) + F'(b)(b_t - b) \right] = \frac{\lambda_t - \lambda}{\lambda}.$$

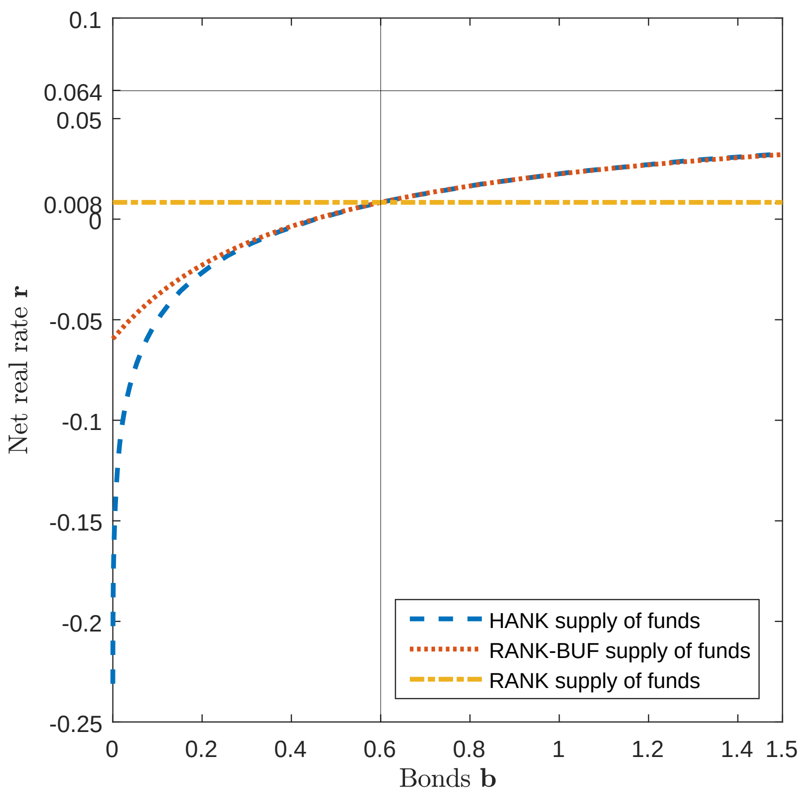
The equation shows that $F(b)$ affects the intertemporal elasticity of substitution as a consequence of the GHH assumption. In order to minimize the impact of this channel on RANK-BUF model fluctuations I choose the benchmark F function parameters in a way that implies $F(b) = 0$.

Table 4 contains the matched parameters of the representative agent models while Figure 2 shows the implied supply of funds curves. Although the curve of the RANK-BUF model is calibrated to be tangent to the HANK's model at the steady state amount of bonds, while at the same time ensuring $F(b) = 0$, Figure 1 shows that the resulting curvature is pretty close to the curvature in the HANK model as well.

Table 4: Calibration of representative agent models.

RANK-BUF		RANK	
Parameter	Value	Parameter	Value
ξ	0.05	β	0.99
b^i	0.4		
ϕ	0.87		

Figure 2: Supply of funds curves



4 Aggregate fluctuations

In this section I consider the responses of the economies to aggregate shocks. The size of the shocks is normalized to imply similar output responses across the shocks in the benchmark specification. I begin the analysis with government shock in the flexible price limit. This setting lets me isolate the impact of additional liquidity provided by the government debt expansion independently from the impact of changes in household risk. I find that the RANK-BUF model provides an excellent approximation to the HANK model in this case.

I next turn to the benchmark specification with sticky prices and consider in turn the monetary policy shock, the government spending shock, and the TFP shock.

4.1 Flexible price limit

Figure 3 presents the government shock in the flexible limit. The target level of government bonds is assumed to increase to 0.9 (a jump of an approximately 50%) which in the long run results in reduced government spending given fixed revenues. In the short run, higher borrowing allows the government to expand which crowds out consumption one-to-one in the flexible price limit (the consumption and government spending changes in levels are mirror images of each other). Given this, household consumption must enter an expanding path, leading to an increase in real rate which must be accommodated by monetary policy, determining the nominal rate and inflation. Generically speaking and given Taylor rule on nominal rate setting, an increase in real rate is not consistent with falling inflation since nominal rates respond to inflation more than one-to-one.

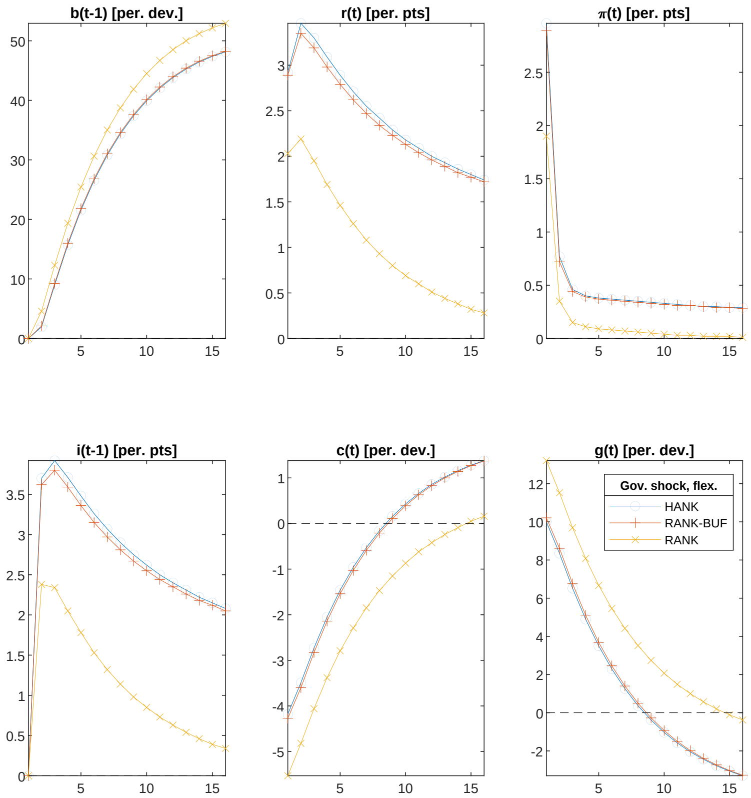
The comparison across models points to the relevance of the upward sloping supply of funds curve. In the long run, the higher liquidity supplied by the government means higher real rates in the HANK model resulting in higher debt service and lower government expenditures relative to the RANK model. As such, consumption grows more, implying larger increase in the real rates. As it turns out, the RANK-BUF model captures this mechanism exceptionally well, not only in the long run but also in the short run.

Note that the difference in the implied real rate response on impact between RANK and RANK-BUF or HANK is substantial, about 1% on impact and rising up to about 2% in the long run.

4.2 The benchmark specification

This subsection evaluates responses to monetary policy shock, government shock, and the TFP.

Figure 3: IRF to a government shock in flexible price limit



Notes: Flexible price limit uses $\kappa = 100$.

The impulse response to a monetary policy shock under the baseline calibration is displayed in Figure 4. The shock is modeled as a one-time transitory disturbance to the nominal interest rate rule. This shock causes an increase in inflation and a corresponding decrease in the nominal interest rate. The inflation increase is however attenuated by the price stickiness, i.e. the price level does not rise as much as in the flexible price limit, inducing larger changes to the real rate. Consumption and government spending both increase in response, although the consumption increase lasts only 2 periods. The corresponding output growth is brought about through the reduction in the labor demand wedge (i.e. an increase in marginal cost), increasing employment and driving down profits.

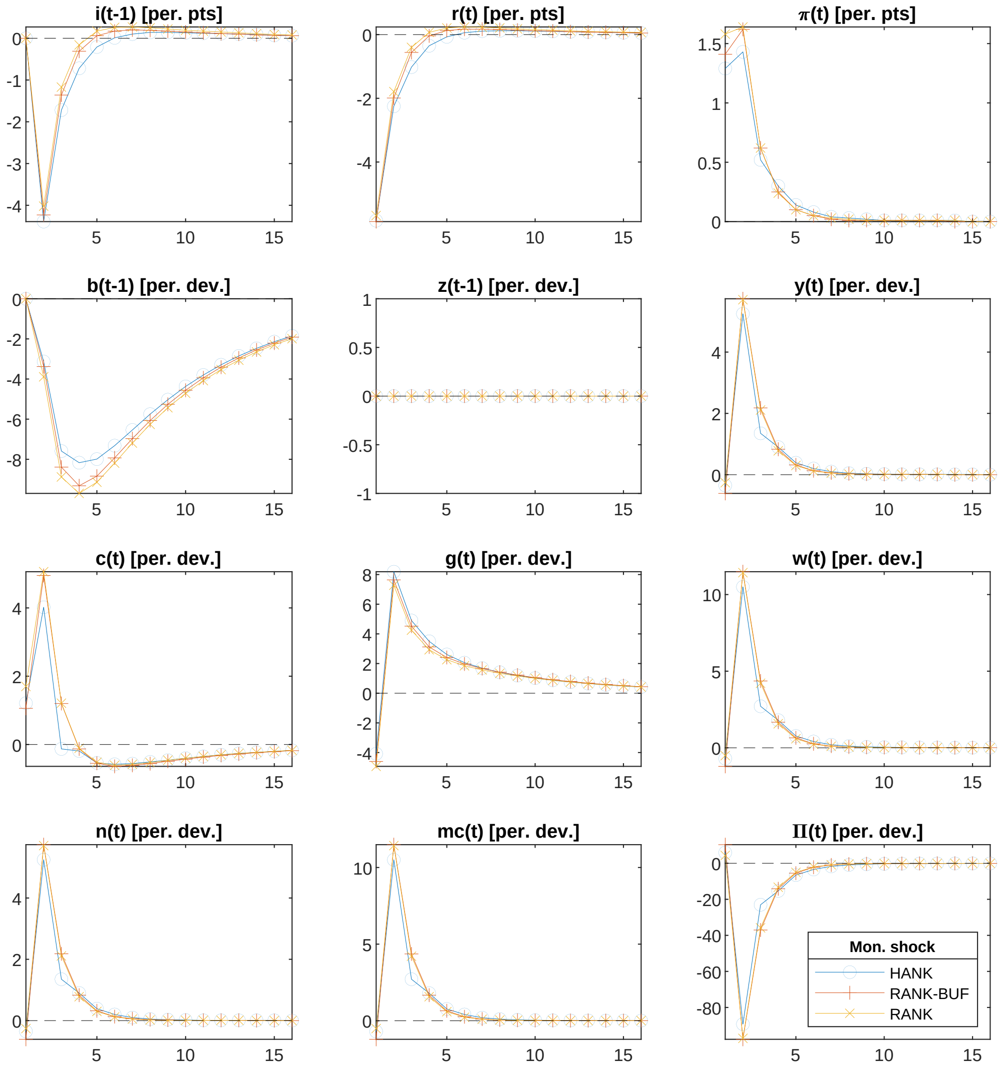
The slight drop in output in the initial period before a change to the nominal interest is caused by the anticipation effect of the shock and it is fairly moderate relative to the actual output increase in the following period.

Regarding cross-model comparison, it does not reveal significant differences between the models. For example, the elasticities of output change to changes in nominal rates are very similar across the models. This is partly a result of small change in bonds and partly due to transitory nature of shocks (households do not desire to change consumption much). This conclusion appears to hold for other transitory shocks as well.

I begin the analysis of persistent shocks with a shock to the bond target level. Here again, with sticky prices the price level is unable to rise as much as would be needed to avoid output changes which results in positive reaction of both consumption and output. The government impact multiplier is about 1.5, i.e. every dollar of government spending induces 1.5 dollars of additional output in the initial period of the shock, which is fairly large. Besides allowing for output changes, price stickiness also induces an initial divergence between the HANK model and the RANK-BUF model (the two models were almost identical in the flexible price case) – the initial consumption response in the HANK model is now about a single percent below that of the RANK-BUF model.

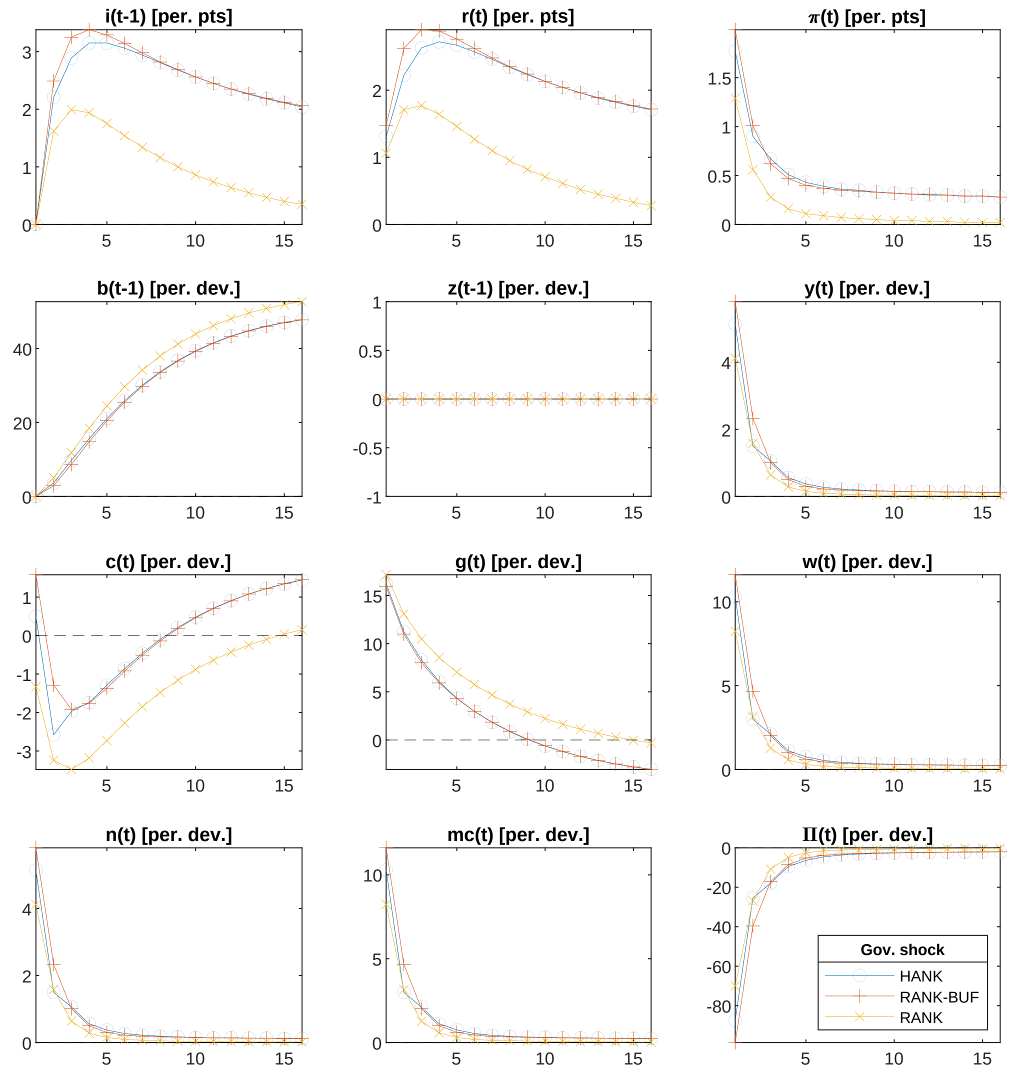
I finally turn to the TFP shock. First of all, the representative agent models display similar responses to each other on impact and they then they diverge from each other after a few periods. This is a consequence of the fall in bonds (the government increases spending and

Figure 4: IRF to a monetary policy shock



Notes: Benchmark calibration.

Figure 5: IRF to a government bond issuance shock



repays some debt) – the models would show almost identical reactions if the government bonds reacted less to inflation and revenues.

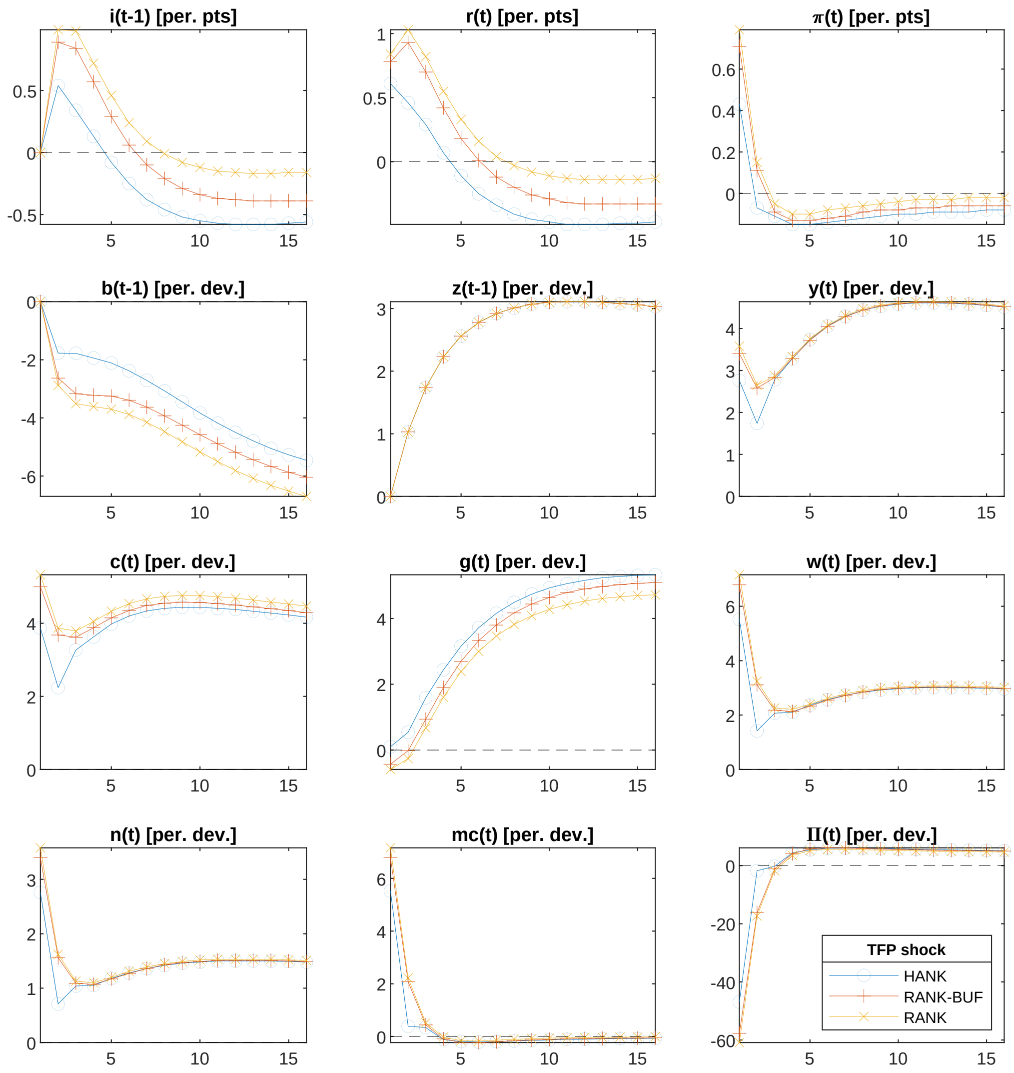
The HANK model on the other hand behaves differently from the other two models. In the long run, the bonds fall by less than in the RANK-BUF case but the real interest rate falls by more – this represents a shift in the supply of funds curve downward. With a rise in productivity, the dispersion in idiosyncratic labor income rises as well which causes the interest rate to fall (households actually desire more bonds for self-insurance at the given real rate relative to the original supply of funds curve). This happens despite an increase in profits which, given the assumption of no dispersion in firm ownership, reduces idiosyncratic risk overall. The shift in the supply of funds curve that is modeled here can therefore be considered as a lower bound on the possible effects.

5 Conclusion

This paper used the methodological shortcut of valued bonds in the representative agent framework to explicitly capture in simple terms the notion of upward-sloping supply of funds curve familiar from the literature on idiosyncratic risk and HANK models. I subjected the models in question to a variety of aggregate shocks in order to closely delineate the impact that upward-sloping supply of funds have on the implied model dynamics. I was able to show that the HANK and the RANK-BUF models indeed behave similarly for a government shock when this shock leaves the risk that households face unchanged. This means that the liquidity enhancing role of the government debt can indeed be well approximated by the assumption of valued bonds.

The RANK-BUF model behaves less satisfactorily when shocks change the profile of risk that households face which I illustrated with the TFP shock. Here the HANK model implied different responses than the two representative agent models not only in the long-run but also in the short-run immediately on impact. Although the RANK-BUF model diverges eventually from the RANK counterpart due to a change in the quantity of government bonds, the initial impulse response is much closer to the RANK model than to the HANK model. This demonstrates that the explicit analysis of risks involved in the aggregate shocks by the HANK model can imply meaningfully different macroeconomic dynamics.

Figure 6: IRF to a TFP shock



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A Determinacy and policy regimes

A.1 Determinacy in the benchmark specification

In this subsection I numerically investigate the issue of determinacy, i.e. the conditions for the existence of a unique bounded equilibrium solution, with the aim of comparing the determinacy regions across models. I start with the benchmark calibration of the models and focus on the parameter θ_{fT} in relation to parameters θ_m and $\theta_{f\pi}$. I then simplify the analysis by assuming flexible prices – this renders the value of parameter θ_{fT} inconsequential because output (and hence government revenues) does not react to endogenous variables in this setting.

Figure 7 shows the main results in the benchmark calibration. The models are in rows, each time in (θ_m, θ_{fT}) space while keeping $\theta_{f\pi}$ at its benchmark value on the left panel and in $(\theta_{f\pi}, \theta_{fT})$ space keeping θ_m at the benchmark on the right.

The regions of determinacy (denoted by red plus signs) in the plots are quite similar across models. This level of granularity shows some less usual properties such as second degree multiplicity, i.e. not enough endogenous government stabilization, in HANK model or second degree of no solution, i.e. too much endogenous government stabilization, in the RANK model. See Farmer and Zabczyk (2019) for the related analysis of higher order indeterminacy in an overlapping generations framework. The models also show some difference in the causes of indeterminacy (i.e. multiplicity or no solution).

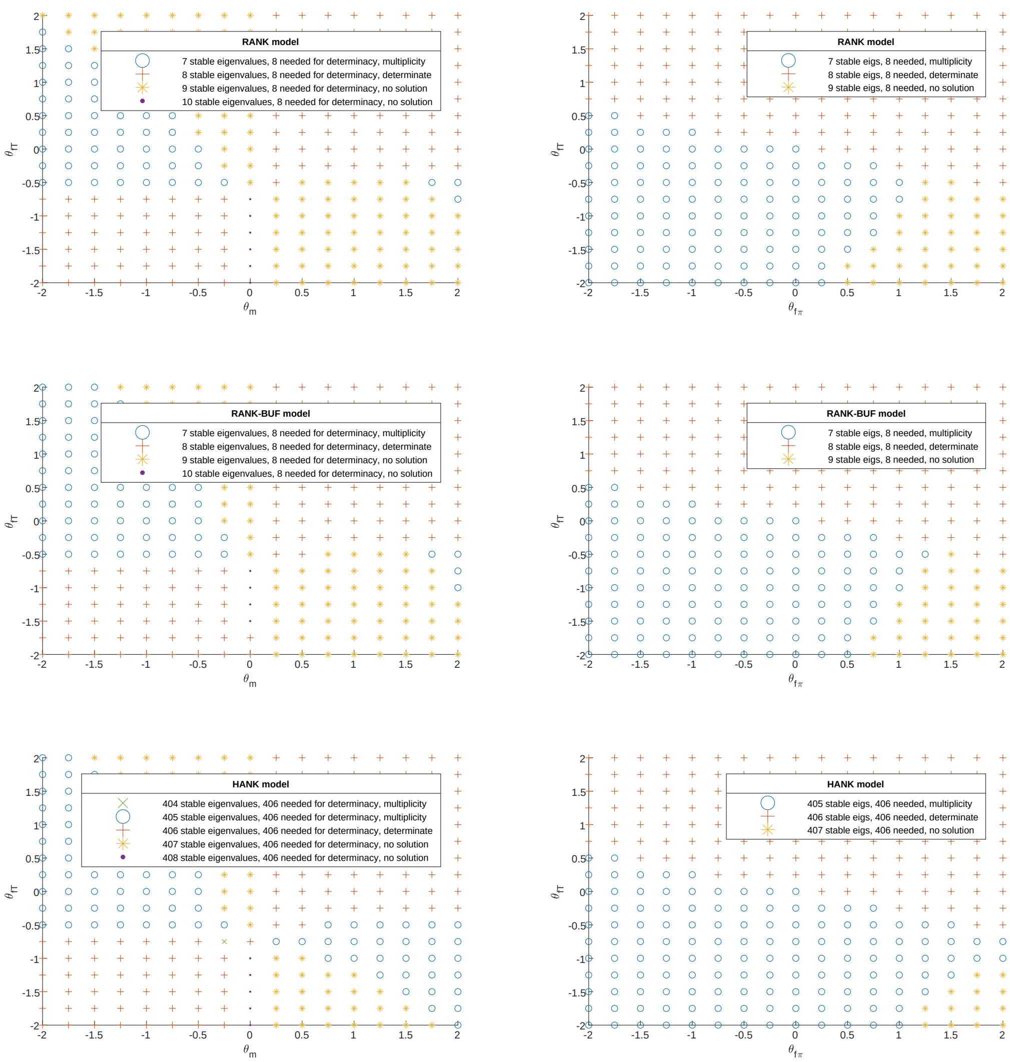
The benchmark calibration values of θ_m and $\theta_{f\pi}$ – falling in the top right area for each model – could be described as corresponding to the case of monetary dominance. Monetary policy increases rates in reaction to elevated inflation (albeit possibly less than one to one) whereas fiscal policy reacts to elevated inflation and revenues by reducing bond issuance and hence government spending. This is reminiscent of the government bond valuation equation of Woodford (1995) – fiscal policy must behave more prudently in times when monetary policy makes debt repayment more difficult due to nominal rate increases. In the other determinacy region, i.e. when monetary policy does not react positively to inflation, fiscal policy must ensure price level determinacy by threatening higher bond issuance in reaction to higher revenue streams.

Fiscal reactions to inflation and to revenues are substitutable – looking at the right panel, stronger reaction to inflation allows for reduced reaction to revenues without losing determinacy (the slope of the determinacy boundary is roughly negative third).

A.2 Determinacy of representative agent models in the flexible price limit

The flexible price equilibrium can be reduced to a simple set of equations in the representative agent case (abstracting from stochastic elements for the moment). After linearization, these

Figure 7: Numerical analysis of determinacy regions in the models



can be written as follows:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-\frac{F''(b)}{(1-F'(b))\sigma}+1-F'(b)+1+r & b-\frac{1}{(1+r)\sigma} & \frac{1}{\sigma}-b(1+i) & F'(b)-1 \\
1 & 0 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix}
b_t-b \\
i_t-i \\
\pi_{t+1} \\
b_{t+1}-b
\end{bmatrix}
=
\begin{bmatrix}
\rho_b & 0 & -\theta_{f\pi} & 0 \\
0 & \rho_i & \theta_m & 0 \\
1+r & b & -b(1+i) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
b_{t-1}-b \\
i_{t-1}-i \\
\pi_t \\
b_t-b
\end{bmatrix},
\tag{21}
$$

where

$$
\sigma=\frac{\gamma}{c-\frac{n^{1+\varphi}}{1+\varphi}+F(b)}.
$$

The equations are, in order, the government bond issuance rule, the monetary policy rule, the Euler equation, and an auxiliary definition of next period bonds. Consumption of the households and government spending level are the 2 remaining non-constant variables, implied by the 2 respective budget constraints.

An interesting aspect of this setup that follows from the Euler equation is mutual interdependence between government bonds on the one hand and inflation on the other. This is one difference relative to the analysis of Leeper (1991) where fiscal and monetary policies decouple – the Euler equation in that framework is a relationship involving current and future inflation rates only (due to the assumption of fixed consumption).

Beside the benchmark case with active monetary policy I also provide an example in Figure 9 where nominal interest rate does not adjust – setting $\theta_m = 0$ and $\rho_b = 1.1$. Due to $\rho_b > 1$, government bonds remain stable only due to offsetting reaction of price level, an example of fiscal dominance. Inflation in this case mirrors real rate movement which jumps up on impact and then falls either to zero for the RANK model or to about 1.5 percentage points above steady state for HANK and RANK-BUF models. This elevated real rate then results in permanent deflation which forces the diverging patter in government bonds (given the fiscal reaction to inflation). As such, the actual increase in bonds is much lower than the new target in the HANK and RANK-BUF cases. The impact of this channel is minimized in the benchmark case as monetary policy ends up forcing the long-run inflation close to zero by raising nominal rates in HANK and RANK-BUF models.

Figure 8: IRF to a government shock in flexible price limit, fiscal dominance.