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Authors: Kai Arvai

Date: January 2021

RTG 2281 – Discussion Papers

No. 2021-1

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Funded by

Deutsche Forschungsgemeinschaft

German Research Foundation

The Political Economy of Currency Unions

Kai Arvai *

University of Bonn

Job Market Paper

January 13, 2021

Abstract

How can a currency union be sustained when member states have an exit option? This paper derives how fiscal and monetary policies can ensure the survival of a common currency, if countries want to leave the union. A union-wide central bank can prevent a break-up by setting interest rates in favor of the country that wants to exit. I show how a central bank does this by following a monetary rule that features time-varying country weights. The paper demonstrates that a central bank can only sustain the union for a while, but not permanently. Fiscal transfers between countries are more effective, as they sustain the currency union also in those situations in which monetary policy fails to do that.

Keywords: Currency union, Monetary policy, Lack of commitment, Exit option, Fiscal policy

JEL Classification: E42, E52, E61, F33, F45

*University of Bonn, Bonn Graduate School of Economics, E-mail address: kai.arvai@uni-bonn.de . I would like to thank my advisors Keith Kuester, Christian Bayer and Benjamin Born for helpful feedback and guidance throughout the project. Furthermore, special thanks goes to Donghai Zhang, Moritz Kuhn and Stephanie Schmidt-Grohé for helpful comments and suggestions. The paper also benefited from participants of the Uni Bonn Macroeconomics seminar, the RGS conference, the NuCamp PhD workshop, the Bonn Mannheim PhD Workshop and the RTG summer school. I also gratefully acknowledge funding and support from the DFG Research Training Group "The Macroeconomics of Inequality"

1 Introduction

Currency unions, such as the eurozone, are inherently unstable. One reason for this instability is the fact that member states are still sovereign nations that can decide to leave the union. The eurozone crisis has forcefully shown this. Next to fiscal policy, the role of the European Central Bank for the currency union has been debated extensively. This poses the question what monetary policy can actually do if the union is confronted with the threat of a break-up. How can a central bank help to make a currency union sustainable?

To address this issue, I set up a two-country open economy model that gives governments the option to choose between being in a currency union and an own national currency. With an own currency, the central bank can focus on price stability and let the exchange rate float freely. In a currency union there is only one central bank for both countries. The benefit of a common currency is that it facilitates trade. By assumption, if both countries use the same currency, trade costs are reduced and bilateral trade increases. The downside of the currency union is that macroeconomic stabilization is less effective for certain states of the world since a common central bank sets interest rates for the whole union. Therefore, the costs of a currency union are time-varying and in some situations these costs might outweigh the benefits.

I use this setup to run an experiment in which I calibrate the economy to simulate and then look at the outcome of four scenarios. In the first scenario both governments decide freely when they want to leave the currency union. That is the only decision. They take monetary policy and the outside option as given. Once a government leaves the currency union, the union is destroyed forever. In the second scenario, I add a union-wide Ramsey planner who sets lump-sum transfers between countries. The planner takes the member states' exit option into account. The idea is to set transfers in such a way that no government wants to leave the union. In the end, under the veil of ignorance, both countries are better off with this transfer scheme as the union is sustained. As in the first scenario, monetary policy is taken as given by the Ramsey planner. The third scenario considers a union-wide central bank, that sets interest rates and takes the exit option of both countries into account. No transfers take place in this scenario. As with the union-wide Ramsey planner, the idea is to set interest rates in such a way that no country wants to leave the union at any point in time. In the fourth and last scenario, I consider a joint monetary and fiscal response with a one-time monetary intervention in the crisis period itself and systematic transfers afterwards. All these four scenarios are run with different amounts of trade gains in a currency union that are consistent with the range of estimates from the literature[1]. The goal is to check which policy works depending on the amount of gains coming from the currency union.

The paper has three main findings: First I show how a central bank can prevent a break-up of the currency union by following an interest rule that puts more weight on stabilizing

[1]See the literature review at the end of the section and the calibration in section 4

crisis countries that would otherwise exit the union. Second, I highlight that interest rate policy alone is a poor tool to redistribute between countries, as it relies on business cycles being not perfectly synchronized. Furthermore, compensation through interest rates is distortionary. Therefore- and this leads to the third result- the central bank alone can only sustain the union for some time, but if a sequence of sufficiently large asymmetric shocks emerges the union will eventually collapse. I demonstrate how fiscal transfers can sustain the union in the experiment in those situations in which interest rate setting alone cannot.

The first finding shows how a central bank can use an interest rate rule to sustain the currency union when member states want to exit. The central bank does this by following a rule that features *time-varying* country weights. When a country wants to leave the currency union, the central bank promises this country to put a greater emphasis on stabilizing its economy. This way the central bank gives more weight to that country and makes the currency union for it relatively more attractive than the outside option with national currencies. Which country is stabilized more by the central bank is determined ex post, after shocks have materialized. Therefore, with the interest rate rule derived in this paper the central bank can in principle factor in exit options of member states.

The second finding relates to the strength of this policy instrument to redistribute and in turn to sustain the union. The central bank can only promise to favor a certain country in the future, if the busines cycles of the member states are not perfectly synchronized. This means that a certain degree of asymmetry between both countries is needed for interest rates to be an effective tool. If business cycles are expected to be perfectly synchronized in the future, the central bank has no way to favor a specific country because both countries want to have the same interest rates. This puts a limit to the ability of the central bank to make promises to countries that are willing to leave, as compared to a planner who can promise transfers.

This leads to the third result, namely that the currency union will eventually break up if monetary policy is the only tool considered to prerserve the union. The experiment shows that an actual break-up of the union is rather likely if fiscal transfers and monetary accommodation are absent. In the simulation, the central bank can increase the average duration of the currency union, but she cannot totally suppress the possibility of a break-up. With a monetary policy intervention, the union can be sustained for a while until a sequence of exceptionally large asymmetric shocks hit the union. I furthermore demonstrate in the experiment that fiscal transfers can sustain the currency union also in those simulations in which monetary policy alone fails to achieve that.

In conclusion, the central bank can help to sustain the union and reduce the probability of a break-up. This is done by partly departing from the original objective of union-wide price stability and emphasizing stabilization of crisis countries. The central bank however is only able to buy some time for the currency union. The option of using fiscal transfers is a more effective policy tool and ensures that the union is permanently sustained.

Related Literature

The first strand of literature that this paper relates to goes back to the optimum currency literature, pioneered by Mundell (1961). Currency unions are vulnerable to so called asymmetric shocks, especially when factor mobility is low and a common fiscal policy is missing, as noted by McKinnon (1963) and Kenen (1969). Eichengreen (1992) and Shambaugh (2012) have discussed if the eurozone constitutes an optimal currency area and noted several vulnarabilities. These vulnerabilities are in fact so large that markets price in a positive probability of a eurozone break-up, as shown by Bayer et al. (2018). My paper explicitly microfounds the costs of a monetary union and models when a break-up occurs. It also discusses how such a break-up can be prevented. I use a two-country model based on Corsetti and Pesenti (2002). This kind of model is part of the new open economy literature that has been established over the last decades [2]. An important issue that this literature addresses is the question which monetary regime is optimal depending on the invoicing regime. Conclusion reach from letting the exchange rate float freely, as proposed by Friedman (1953) and Clarida et al. (2002), to pegging the exchange rate as in Devereux and Engel (2003). Optimal cooperation between monetary authorities has also been extensively discussed by the literature, see Benigno and Benigno (2003), Corsetti and Pesenti (2002), Corsetti et al. (2018), Bodenstein et al. (2019) and Egorov and Mukhin (2020). Historically, the world has seen many different exchange rate regimes, as shown by Ilzetzki et al. (2019). How exchange rate regimes are chosen and why they evolve in the way we observe it, is not well understood and has been discussed recently by Mukhin (2018). I contribute to this literature and show under which conditions a currency union, seen as a fixed exchange rate regime, can collapse and be sustained. Why such unions are formed in the first place is an open debate. My paper considers trade advantages in a currency union as the main benefit, as a common currency is thought to reduce trade costs (Alesina and Barro, 2002). Evidence of more trade inside a currency union has been given by Baldwin et al. (2008) and Micco et al. (2003) who find trade increases between 4% to 16%. Even higher estimates have been found by Rose (2000), Frankel and Rose (2002) and Glick and Rose (2002). Baier et al. (2014) highlight that those large increases in bilateral trade of economic unions arise if other economic trade agreements such as customs union and common markets are considered as well. Another potential benefit of entering a currency union is the reduction of inflationary biases in some countries, when a new credible central bank is created, see for example Alesina and Barro (2002). A similar point has been made by Chari et al. (2020) who points out that an inflationary bias can be reduced in a currency union even if the newly created central bank is not credible. My paper therefore combines the good and bad sides of a currency union in one model. The costs of the currency union are time-varying and might exceed the benefits when a big asymmetric shock emerges. Such a situation gives rise to the possibility of a break-up of

[2] See for example Benigno and Benigno (2003), Gali and Monacelli (2005) Clarida et al. (2002), Corsetti and Pesenti (2005),Corsetti et al. (2011) and Engel (2011)

the union, that is discussed in the second part of the literature review.

Forming and disrupting political and economic unions has been analyzed by Balassa (1961), Haas (1958) and Bolton and Roland (1997). As noted by Cohen (1993), a currency union consisting out of sovereign nations can break up. Fuchs and Lippi (2006) formally establish an exit option in a reduced-form model of a monetary union. They embed this union into a dynamic contract with limited commitment[3] of member states to the union. They find that with such an exit option, the union-wide central bank optimally uses time-varying country weights. I contribute to that by explicitly modeling the macroeconomics of a currency union and deriving an interest rate setting rule that features time-varying country weights as well. In addition to that, I compare this policy to fiscal transfers that aim to make the currency union sustainable. Auclert and Rognlie (2014) show that a monetary union can favor the creation of a fiscal union. They demonstrate how a central bank departs from its traditional role of price stability for the union to encourage more political integration with its policy. In a similar way, my paper shows how a central bank can prevent political *dis*integration with its policy. Ferrari et al. (2020) have demonstrated how fiscal policy can be used as a tool to deal with exit options and significantly reduce the costs of a currency union. Compared to them, I introduce aggregate risks and provide a framework to jointly analyze fiscal and monetary policy. How fiscal policy can improve welfare in a currency union has been shown by Farhi and Werning (2017). They establish that even in the presence of perfect financial markets, as in Cole and Obstfeld (1991), fiscal policy plays an important role in stabilizing a currency union. Recently, the literature discussed fiscal policy in the context of moral hazard in Europe, see for example Ábrahám et al. (2019) and Müller et al. (2019). In my paper, fiscal policy can improve the outcome by ensuring that governments do not exert the exit option. This way, the currency union is sustained and both countries benefit from trade costs over a longer horizon. Other papers consider exit options as well, such as Kriwoluzky et al. (2019) who find that a sovereign debt crisis can be amplified by exit expectations, or Eijffinger et al. (2018) highlighting crisis contagion to other member states in the presence of exit options. Another result of my paper relates to political integration more generally. I show how countries decide to join a currency union with no transfers in the beginning. As the threat of a break-up looms, both countries voluntarily enter a primitive fiscal union with transfers between countries. The threat of a break-up serves as a driver of a deeper political and economic union, since countries automatically climb the 'staircase' of political integration, as in Auclert and Rognlie (2014).

The work is organized as follows. Section 2 presents the two-country model that gives rise to different monetary regimes and the benchmark allocations. In section 3 I describe the political economy, where governments choose the monetary regime. Section 4 discusses the calibration of the model, while section 5 runs the experiment and shows the results. Section 6 concludes.

[3]The literature of dynamic contracts with commitment problems was pioneered by Thomas and Worrall (1988), Kocherlakota (1996) and Marcet and Marimon (2019).

2 Model of the Economy

This section outlines a model based on Corsetti and Pesenti (2002), and Corsetti and Pesenti (2005). I establish a dynamic two-country general equilibrium model with trade and stochastic productivity shocks. I extend the baseline by Corsetti and Pesenti (2002) to allow for trade costs and to explicitly give the governments the option to choose between a currency union and national currencies.

2.1 Households, Consumption Bundles and Price Indices

There are two countries, a Home country (H) and a foreign country (F). Each is populated by a mass one of identical individuals. Lifetime utility of the representative household in H is given by:

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty}\beta^{j-t}\left(\ln(C_j)-\kappa L_j\right)\right] \tag{1}$$

where C_t is a basket of consumption goods and L_t are working hours for the individual with κ being a coefficient for disutility of labor. $\beta \in (0,1)$ is the time discount factor which is assumed to be the same for individuals in both countries. In addition to that, utility is quasi-linear in labor to simplify the aggregation in later steps. Preferences of agents in F are described analogously with all variables being denoted with a *. The consumption basket consists of consumption of Home goods $C_{H,t}$ and foreign goods $C_{F,t}$ with an elasticity of substitution of 1. It can be written as a Cobb Douglas function:

$$C_t = (C_{H,t})^{\gamma}(C_{F,t})^{1-\gamma}, \quad C_t^* = (C_{H,t}^*)^{1-\gamma}(C_{F,t}^*)^{\gamma} \tag{2}$$

where γ governs the taste of households for goods from country H or F. In contrast to Corsetti and Pesenti (2002), I assume that both countries have a Home bias and that every country weights its own good with γ. The individual's consumption index for goods from country H is an aggregator of different brands h with elasticity of substitution θ:

$$C_{H,t} = \left[\int_0^1 C(h)^{\frac{\theta-1}{\theta}}dh\right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[\int_0^1 C(f)^{\frac{\theta^*-1}{\theta^*}}df\right]^{\frac{\theta^*}{\theta^*-1}} ; \quad \theta,\theta^* > 1$$

Each country hence specializes in the production of a single type of good. Each brand h is produced by a single Home firm and sold in all countries in a monopolistic market. The utility-based price index P_t of H is the consumption-based price index that can be obtained by minimizing expenditures to buy one unit of composite real consumption C_t.

$$P_t = \frac{P_{F,t}^{1-\gamma}P_{H,t}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \quad P_t^* = \frac{P_{F,t}^{*\gamma}P_{H,t}^{*1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}.$$

The price indexes $P_{H,t}$ and $P_{F,t}$ for Home goods and foreign goods respectively in the Home country can be derived in a similar way:

$$P_{H,t} = \left(\int_0^1 p_t(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}, \quad P_{F,t} = \left(\int_0^1 p_t(f)^{1-\theta^*} df \right)^{\frac{1}{1-\theta^*}}.$$

Household's portfolio consists of several components. Agents can access financial markets in order to sell and buy Home bonds[4] $B_{H,t}$ and foreign bonds $B_{F,t}$. Foreign bonds have to be converted into Home currency. The exchange rate $\mathcal{E}_t$ is defined as Home currency over foreign currency [5]. In addition, the households own the firms and supply labor on a competitive market. Therefore, they receive wages, firms' profits $\Pi_{H,t}$ and interest rates from bonds. Furthermore, they pay non-distortionary net taxes T_t to the government. As in Woodford (2003), I consider the limiting case of a cashless economy. The nominal flow budget constraint of individual j at time t is given by the following inequality:

$$B_{H,t}+\mathcal{E}_t B_{F,t}+P_{H,t}C_{H,t}+P_{F,t}C_{F,t}+T_t \leq (1+i_t)B_{H,t-1}+(1+i_t^*)\mathcal{E}_t B_{F,t-1}+W_t L_t+\Pi_{H,t} \tag{3}$$

the short-term nominal interest rate i_t is paid out at the beginning of period t and known in $t-1$. The household's optimization problem is to maximize lifetime utility (1) subject to the consumption aggregator (2) and the budget constraint (3). Demand for brand h and f by the representative consumer can then be expressed as a function of the relative price and total consumption of Home and foreign goods:

$$C_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}, \quad C_t(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\theta^*} C_{F,t} \tag{4}$$

Consumption of goods produced in the Home country is a function of its price relative to the overall price index and total consumption:

$$C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t, \quad C_{F,t} = (1-\gamma) \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t$$

Demand can also be expressed as a function of international relative prices. Let the terms of trade $\mathcal{T}_t$ be defined as the price of foreign export goods over Home export goods.

$$\mathcal{T}_t = \mathcal{E}_t P_{F,t}^* / P_{H,t}. \tag{5}$$

The Euler equation determines agent's intertemporal allocation

$$\frac{1}{P_t C_t} = (1+i_t)\mathbb{E}_t \left[\beta \frac{1}{P_{t+1} C_{t+1}} \right] \tag{6}$$

[4] $B_{H,t-1}$ are accumulated bonds until the period t that are carried over to period t. Households choose in t how many bonds to hold.

[5] A higher $\mathcal{E}_t$ means that one unit of a foreign currency can now buy more units of the Home currency. We say the Home currency depreciates.

The stochastic discount factor is defined as $Q_{t,t+1} \equiv \beta \frac{P_t C_t}{P_{t+1} C_{t+1}}$. The optimality condition for labor $W_t = \kappa P_t C_t$ implies that $Q_{t,t+1}$ is the same for every individual. In addition, the law of one price holds. Thus $p_t(h) = \epsilon_t p_t(f)$.

2.2 Production, Good Transport and Prices

Production in the model is a function of labor input and a stochastic technology parameter a_t. Supply of brand h is given by

$$Y_t(h) = L_t(h) a_t \,. \tag{7}$$

The technology parameter determines aggregate productivity in the economy and is the only source of uncertainty in the model. a_t and its foreign analog a_t^* follow an identical stochastic process. Let $s_t = (a_t, a_t^*)$ denote the state of the world. a_t is a random variable with support A, its history is described by $s^t = (\{a_t, a_t^*\}, \{a_{t-1}, a_{t-1}^*\}, ..., \{a_0, a_0^*\})$. The process is Markov with transition matrix $p(s^t)$. Higher values of a_t correspond to greater productivity ('boom') while lower values indicate lower productivity ('recession'). In this setup, one country can be in a boom, while the other is in a recession. Such a state is considered as an asymmetric shock.
A firm faces demand for brand h by consumers in H and in F, as given by (4). Total demand for firm h is

$$\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} + (1+\varpi)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\theta} C_{H,t}^*. \tag{8}$$

At this point, I extend the model and assume that a certain fraction ϖ of goods in the non-domestic market are lost. Like in Alesina and Barro (2002), iceberg trade costs occur when transporting a good to the non-domestic market. It is necessary to ship $1+\varpi$ units from H to F if one unit of h shall arrive in F. Crossing the border between two countries entails transport costs reflecting for example currency conversion costs. These expenses are lost for the economy. I assume that the adoption of a common currency reduces these costs. For the calibration in section 4, a range of empirical estimates from the literature discipline ϖ.
Labor markets are competitive. Let W_t denote the nominal wage. Nominal marginal costs are identical across firms:

$$MC_t(h) = MC_t = a_t^{-1} W_t$$

Profits generated in the foreign market need to be converted into the Home currency. The firm knows that a certain fraction of goods is lost when selling them in the non-domestic market. Knowing overall demand (8), profits are given by

$$\Pi_t(h) = \Big((1-\tau)p_t(h) - MC_t\Big)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} + \Big((1-\tau)\mathcal{E}_t p_t^*(h) - (1+\varpi)MC_t\Big)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\theta} C_{H,t}^*. \tag{9}$$

$p_t(h)$ is the nominal price of brand h in H and $\mathcal{E}_t$ is the nominal exchange rate between both countries defined as units of Home currency per unit of foreign currency. $p_t^*(h)$ is the price of brand h in the foreign market. As in Benigno and Benigno (2003), τ is a country specific proportional tax on firms' revenues that is rebated to households via lump-sum transfers. This tax eliminates monopolistic markups.

The model features nominal rigidities: Firms set prices $p_t(h)$ one period in advance, in $t-1$. They form expectations about productivity in the next period and maximize the present discounted value of profits. For given prices, firms satisfy demand for their good[6]. Firms optimally set prices equal to expected marginal nominal costs multiplied with the equilibrium markup Φ.

$$p_t(h) = P_{H,t} = \Phi \mathbb{E}_{t-1}[MC_t] \tag{10}$$

where Φ is the level of monopolistic markup corrected by distortionary taxation:

$$\Phi = \frac{\theta}{(\theta-1)(1-\tau)}, \quad \Phi^* = \frac{\theta^*}{(\theta^*-1)(1-\tau^*)}$$

$\frac{\theta}{\theta-1}$ is the markup that arises due to monopolistic competition. For $\Phi = 1$, monopolistic distortions are completely eliminated by taxes. If they are not completely eliminated Φ is greater than 1 and makes prices greater than their marginal costs.

Firms selling abroad also set their prices one period in advance. I assume that these prices are set according to the Producer Currency Pricing (PCP) model. This means that exported goods are sold in the currency of the producer. For example, goods produced in H and sold in F are priced in H's currency. The price firms receive from selling goods to a foreign country is not affected by exchange rate movements. For given quantities, exchange rate variations have no impact on profits, because prices move one to one. For consumers however, the price of non-domestic goods depends on the exchange rate. Let the price for exports that firms choose in their currency be denoted by $\tilde{p}_t(h)$. The actual price that consumers face in their currency is $p_t^*(h)$. Both prices are linked via the exchange rate:

$$p_t^*(h) = \frac{\tilde{p}_t(h)}{\mathcal{E}_t}$$

[6] Firms only sell goods, if their prices is higher than the marginal costs, that is $P_{H,t} \geq MC_t$ and $P_{H,t}^* \geq \frac{MC_t}{\mathcal{E}_t}(1+\varpi)$ Firms that do not met the participation constraint will not sell goods. I only look at versions of the model, where prices are higher than marginal costs.

Firms choose the price of their export goods $\tilde{p}_t(h)$ such that their profits (9) are maximized.

$$p_t^*(h) = P_{H,t}^* = \Phi(1+\varpi)\frac{\mathbb{E}_{t-1}\left[MC_t\right]}{\mathcal{E}_t} \tag{11}$$

The transportation costs ϖ increase prices of h in F. Prices of foreign brands in country H are analogous:

$$p_t(f) = \tilde{p}_t(f)\mathcal{E}_t$$

For foreign goods we have

$$P_{F,t}^* = \Phi^*\mathbb{E}_{t-1}[MC_t^*], \quad P_{F,t} = \Phi^*(1+\varpi)\mathcal{E}_t\mathbb{E}_{t-1}[MC_t^*] \tag{12}$$

2.3 Government and Central Bank

The government runs a balanced budget every period.

$$T_t = \tau p_t(h)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} + \tau\mathcal{E}_t p_t^*(h)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\theta} C_{H,t}^*$$

The model also features a central bank that controls the interest rate i_t and provides a nominal anchor for market expectations. Furthermore, the central bank has an inflation target Π. Inflation Π_t is defined as

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

Monetary policy can be useful by closing output and employment gaps in the presence of price stickiness. The central bank uses interest rates to operate via the Euler equation. As in Corsetti and Pesenti (2002), I introduce a monetary stance $\mu_t = P_tC_t$ that controls nominal expenditures in the economy. This stance links the nominal interest rate in the Euler equation such that

$$\frac{1}{\mu_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1}{\mu_{t+1}}\right]$$

μ_{t+1}/μ_t determines inflation Π_t, the steady state nominal interest rate is $1+i = \Pi/\beta$. In equilibrium one obtains that $\mu_t = P_tC_t = W_t/\kappa$[7]. An expansionary monetary policy in H corresponds to interest rates cuts today or households' expectations about interest rate cuts in the future. In this case μ_t lies above the trend, it coincides with increased nominal spending P_tC_t in the economy.

[7]Inspect the Euler equation with logarithmic utility for that

2.4 Market Clearing

The labor market in H and F is cleared:

$$L_t = \int_0^1 L_t(h)dh, \quad L_t^* = \int_0^1 L_t(f)df$$

International financial markets for bonds are cleared, all bonds are in zero net supply:

$$B_{H,t} + B_{H,t}^* = 0, \quad B_{F,t} + B_{F,t}^* = 0$$

Supply of each brand (7) equals its aggregate demand (8)

2.5 Benchmark Allocations

This section discusses monetary policy in a currency union and with national currencies. I derive the allocation of consumption and labor in those two regimes with sticky prices.[8] In section 3, the governments will choose between these two regimes.

2.5.1 National Currency

Consider a central bank that commits to pre-announced rules in country H. The national authority in the Home country chooses its monetary stance μ_t and maximizes expected utility of the representative agent. The central bank takes the information set of last period as given. As in Corsetti and Pesenti (2005), the central bank of H does not resort to time-inconsistent discretionary monetary policies, rather it acts under commitment:

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \left[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^0) \Big(\ln(C_t) - \kappa L_t \Big) \right]$$

The problem is subject to the equilibrium conditions of the economy. For further details, see section A.7.1. The optimal policy of the central bank ensures price stability:

$$MC_t = \mathbb{E}_{t-1}[MC_t] \tag{13}$$

This means, that the central bank chooses interest rates in such a way, that actual marginal costs for domestic firms always equal expected marginal costs. With this policy, the central bank replicates the flex-price equilibrium[9] and eliminates any distortion coming from rigid prices. This implies that monetary policy is completely inward looking. The central bank stabilizes the domestic price index only. As noted by Benigno and Benigno (2003) this is a very special result and relies on the PCP assumption and that the trade elasticity of substitution (Cobb Douglas aggregator) as well as the intertemporal

[8] The benchmark allocations of a social planner is discussed in the Appendix A.3.1, as well as the allocation in an economy with flexible prices Appendix A.6.

[9] see A.6

elasticity (log consumption) are both set to 1. An inward-looking monetary stance also means that only domestic productivity shocks are considered, and the central bank does not want to manipulate the terms of trade.

Consider an example: In one period, productivity in H is higher than previously expected. This means that marginal costs of home firms fall. In the presence of price stickiness, prices cannot fall in the same period. This means that prices of home goods are too high, implying inefficiently low demand for home goods. Optimal monetary policy cuts interest rates in such a situation. This boosts nominal expenditures of the economy and causes the exchange rate of the home country to depreciate. As the home currency gets cheaper, foreign households can now buy more home goods with their own currency. The exchange rate movement mimics the price fall that would have occurred in a flexible price world. This way, domestic and foreign demand is put to its efficient flex price level. With this policy in place, actual marginal costs always equal their expected value, implying price stability for the whole economy.

The central bank in the Foreign country operates in the same way as in the Home country. The optimal policy of the central bank in F implies price stability for F and is completely inward looking as well. As a result, the exchange rate is flexible.

With both central banks following their policy rules, I can analytically compute consumption and labor as in Corsetti and Pesenti (2002). These variables have the superscript 'N' for national.

$$\begin{aligned} C_{Ht}^{N} &= \frac{\gamma a_t}{\Phi\kappa} & C_{Ht}^{*N} &= \frac{(1-\gamma)\left(\frac{1}{1+\varpi}\right)a_t}{\Phi\kappa} \\ C_{Ft}^{N} &= \frac{(1-\gamma)\left(\frac{1}{1+\varpi}\right)a_t^*}{\Phi^*\kappa^*} & C_{Ft}^{*N} &= \frac{\gamma a_t^*}{\Phi^*\kappa^*} \\ L_t^N &= \frac{1}{\Phi\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) & L_t^{*N} &= \frac{1}{\Phi^*\kappa^*}\left(\frac{\gamma}{1+\varpi} + 1 - \gamma\right) \end{aligned} \tag{14}$$

Consumption moves together with productivity, while labor does not, as in the efficient allocation of the social planner (A.1). Trade costs ϖ decrease consumption and employment and cannot be eliminated by the central bank. There is also no other state variable, such as wealth. As in Corsetti and Pesenti (2002) the current account is always balanced and households of a country do not accumulate any debt or wealth. The reason for that is that endogenous terms of trade movements offset productivity shocks, if the inter- and intratemporal elasticity of substitution are both set to 1. For further details, see section A.1. I also consider the possibility of a non-credible central bank in F that is not able to commit to any policies. If such a central bank is in charge, an inflationary bias can arise. The policy problem and the implied allocation is described in A.7.4. For now, we focus on a situation in which both central banks can commit to policies as the main benchmark.

2.5.2 Currency Union

In a currency union, monetary policy is conducted by a union-wide central bank that sets interest rates for the whole union. I assume that there are no trade costs in a currency union, as both countries use the same currency. The central bank of the union maximizes the weighted sum of both countries' representative agents' lifetime utility. Let ξ be the weight for country H and $1-\xi$ be the weight for country F. The objective function for the union-wide central bank is

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \left[\xi \sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^0) \Big(\ln(C_t) - \kappa L_t \Big) + (1-\xi) \sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^0) \Big(\ln(C_t^*) - \kappa^* l_t^* \Big) \right]$$

subject to the equilibrium conditions in a currency union, see section A.7.2. Price stability is the optimal policy, as the central bank stabilizes the weighted average of both countries' marginal costs.:

$$1 = \left(\big(\xi\gamma + (1-\xi)(1-\gamma)\big) \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \big(\xi(1-\gamma) + (1-\xi)\gamma\big) \frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} \right)^{-1} \tag{15}$$

Let $\Psi = \big(\xi\gamma + (1-\xi)(1-\gamma)\big)$ and $1 - \Psi = \big(\xi(1-\gamma) + (1-\xi)\gamma\big)$ be the effective weights in front of marginal costs. To illustrate the intuition for this monetary rule, consider the case in which there is no Home bias ($\gamma = 0.5$), e.g. Home and foreign goods are equally important to all. In that case, the effective weight is 0.5 as well, independent from the Pareto-weight ξ. As both countries like both goods in the same way, the central bank also stabilizes both marginal costs in the same way and Pareto-weights are irrelevant. Another interesting case is the scenario in which there is an equal weight $\xi = 0.5$ for both countries. In this case, the effective weight is 0.5 as well. The central bank has to stabilize both countries equally, as both are equally important to the central bank and both have a *symmetric* Home bias to their own goods.[10] Note, that if the weight for the Home country is 1 ($\xi = 1$), the effective weight in front of the Home country's marginal costs equals γ. The effective weights for marginal costs are therefore in line with Home's weight for the corresponding goods in its own consumption bundle. For $\xi = 0$, the effective weights for Foreign marginal costs would be γ, in line with Foreign's taste for Foreign goods. In section 3, I derive how these effective weights become state dependant when there are exit options and how the central bank can use this to favor a specific country of the union to prevent it from exiting.

When both countries have the same productivity, monetary policy coincides with the policy, that a national central bank would have chosen in (13). Actual and expected marginal costs are the same for both countries in that case. If there is an asymmetric shock, optimal union-wide monetary policy differs from national monetary policy. If one

[10] This would not be the case, if both countries have different Home biases. For example, if Foreign has a Home bias γ^*, the effective weight in front of Home marginal costs would be $0.5\gamma + 0.5(1-\gamma^*)$.

country experiences a boom with high productivity and the other a recession with low productivity, the central bank only stabilizes the economy on average.
Consumption and labor in a monetary union have the superscript 'U' for Union.

$$\begin{aligned}
C^{U}_{Ht} &= \frac{\gamma(\Psi a_t^{-1}+(1-\Psi)a_t^{*-1})^{-1}}{\Phi\kappa} & C^{*U}_{Ht} &= \frac{(1-\gamma)(\Psi a_t^{-1}+(1-\Psi)a_t^{*-1})^{-1}}{\Phi\kappa} \\
C^{U}_{Ft} &= \frac{(1-\gamma)(\Psi a_t^{-1}+(1-\Psi)a_t^{*-1})^{-1}}{\Phi^*\kappa^*} & C^{*U}_{Ft} &= \frac{\gamma(\Psi a_t^{-1}+(1-\Psi)a_t^{*-1})^{-1}}{\Phi^*\kappa^*} \\
L^{U}_{t} &= \frac{1}{\Phi\kappa}\frac{a_t^{-1}}{\Psi a_t^{-1}+(1-\Psi)a_t^{*-1}} & L^{*U}_{t} &= \frac{1}{\Phi^*\kappa^*}\frac{a_t^{*-1}}{\Psi a_t^{-1}+(1-\Psi)a_t^{*-1}}
\end{aligned} \tag{16}$$

The amount of labor does depend on productivity in the monetary union. Agents of the recession country work more than agents in the boom country, since the central bank is not able to close all output and employment gaps. As a result, utility of agents in the recession country is lower than in the boom country, making it more attractive in a recession to leave the monetary union. As with national currencies, there is no other state variable, as households do not accumulate any debts or wealth. In a currency union, labor adjusts as a response to productivity shocks in such a way, that it offsets the movement in productivity. Production and consumption for both countries are always the same, implying a balanced current account and no debts dynamic. Note that labor is at the (efficient) flex-price level when productivity is the same for both countries. Trade costs are completely eliminated in a currency union.

3 Political Economy in a Currency Union

Consider now the political economy of currency unions. The goal is to model the decision process of a break-up of a currency union. I model the currency union as a dynamic contract, that each government is free to walk away from. This is based on Ligon et al. (2002) and draws from work by Ljungqvist and Sargent (2004), chapter 18-20 and Thomas and Worrall (1988).
Suppose both countries are initially in a currency union. In every period, the governments of both countries decide if they want to leave. That is the only decision of the government. They base this decision on lifetime utility of the representative agent in the country given a certain state today. The allocation in the corresponding regimes are taken as given. If a representative agent is better off in a currency union than with national currencies, the government decides to stay in the union. This is the case if utility as a function of consumption and labor (16) in a currency union plus the continuation value of the union is higher than utility with national currencies (14). In contrast, a country leaves the union if an agent obtains higher lifetime utility with national currencies. In this case the *participation constraint* of the country is violated. I assume, that once a government has decided against a currency union, no further currency union can be formed in the future and everyone keeps national currencies for the rest of the time.

I use this to set up the scenarios discussed in the introduction: First I consider a model environment in which both countries start with a common currency and the governments decide in each period if they leave the union. After that, I discuss a union-wide Ramsey planner with transfers who takes the lack of commitment of both countries into account. In a next step, I discuss if a central bank with interest rate setting only is able to sustain the union as well. Last I consider interest rates and transfers combined.

3.1 National Social Planner with Exit Option

The monetary union is modeled as a contract that both governments are free to walk away from whenever they want to. The history s^t summarizes past and present shocks and -conditional on the model- monetary regimes. Let $u^i(s^t) = \ln(C^i(s^t)) - \kappa L^i(s^t)$ denote the period utility of country H and $v^i(s^t)$ the corresponding per period utility of country F in regime $i \in \{N, U\}$ for history s^t. Consumption and labor are as in the allocation of (16) for the union and as in (14) with national currencies. The utility gain from a monetary union over national currencies from period t onward is defined as

$$U_t(s^t) = u^U(s^t) - u^N(s^t) + \mathbb{E}_t\left[\sum_{j=t+1}^{\infty} \beta^j \big(u^U(s^j) - u^N(s^j)\big)\right] \tag{17}$$

The first term is the short-run gain from the union and the last term in expectation the long-run continuation gain from the union. The utility gain $V_t(s^t)$ for country F is defined in an analogous way.

From an economic perspective, a national planner (for example the government) decides to leave the union as soon as the expected utility gain of the representative agent is negative. When this happens, the monetary union breaks up, even if the other country has a positive gain. More formally, a government has no incentive to leave the union, if

$$U_t(s^t) \geq 0, \quad V_t(s^t) \geq 0. \tag{18}$$

These two participation constraints are central for the political economy of currency unions. An allocation in a currency union is said to be *sustainable*, if both inequalities hold. Whether they hold, depend on the specific history s^t that summarizes: The current state of the economy, how volatile the economy is expected to be and the transfer history in the contract. Remember that in a monetary union, the central bank struggles to effectively stabilize output if an asymmetric shock occurs. The more asymmetric the shock is, the larger is the welfare loss in a monetary union. With these participation constraints, the allocation of the national social planner can be computed for any sequence of shocks. Before doing this, let us compare this to a union-wide social planner with transfers.

3.2 Union-wide Social Planner with Transfers amid Exit Option

In a next step, I consider a union-wide planner that sets transfers (the Ramsey planner) taking the lack of commitment from member states into account. Therefore, the contract also includes transfers between countries. These transfers correspond to the lump-sum transfers in the two-country model before, see (3). A contract $T(\cdot)$ now specifies for all histories s^t a transfer $T(s^t)$ from H to F. Consumption in a monetary union is therefore $C^U(s_t) - T(s^t)$ for H and $C^{*U}(s_t) + T(s^t)$ for F. Let $u^i(s^t) = \ln(C^i(s_t) - T(s^t)) - \kappa L^i(s_t)$ denote the period utility of country H and $v^i(s^t)$ for F as before that include transfers. If transfers are always zero, the situation is the same as in the allocation of a national social planner in section 3.1. To solve for optimal transfers, it is helpful to consider the Markov structure of the problem. The optimization problem of finding an efficient contract is always the same, when the same state occurs. Furthermore, an efficient contract has after every history s^t an efficient continuation contract. As both participation constraints are therefore forward-looking, the set of sustainable continuation values depends only on the current state of the world. The challenge therefore is that the optimization is subject to forward looking and occasionally binding constraints (the participation constraints). A tool for solving this model is the promised utility approach. By introducing an additional state variable, promised utility, the planner obtains a policy instrument to solve this problem.[11] To get all efficient contracts, the Pareto frontier and its domain of definition must be known. This depends on the convexity of the set of sustainable allocations and the set of sustainable discounted surplus. It can also be shown that the set of sustainable surpluses is a compact interval $[\underline{U}(s^t), \bar{U}(s^t)]$ for H and for F $[\underline{V}(s^t), \bar{V}(s^t)]$, see Appendix A.12.4. The minimum surplus is $\underline{U}(s^t) = 0$, meaning that a currency union and national currencies yield the same utility.

Next define $V(s^t, U(s^t))$ to be the ex post Pareto frontier which solves the following problem: Maximize F's surplus discounted to period t by choosing a transfer today $T(s^t)$ for state s^t and making state-contingent promises about future utility $U(s^{t+1})$. This problem is subject to giving H at least $U(s^t)$. $U(s^t)$ is promised utility in state s^t that was given by the planner to the country H in the period before. Since the new contract chosen at state s^t must be sustainable, both participation constraints are required to be satisfied for all future states s^{t+1}. Thomas and Worrall (1988) show that the Pareto frontier is decreasing, strictly concave and differentiable on the interval. This will also be the case here. It can also be shown that the constraint $U(s^{t+1}) \leq \bar{U}(s^{t+1})$ is equivalent to $V(s^{t+1}, U(s^{t+1})) \geq \underline{V}(s^{t+1})$. The bounds of the interval and the relationship between V and U are intuitive: When H receives the maximum surplus $\bar{U}(s^{t+1})$ of the union in state s^{t+1} , F must receive the minimum surplus of the union in state s^{t+1}, $\underline{V}(s^{t+1}) = V(s^{t+1}, \bar{U}(s^{t+1}))$. If that is not fulfilled, one could either lower or increase one country's surplus and still have a sustainable contract.

[11] Marcet and Marimon (2019) sideline the promised utility approach by studying a recursive Lagrangian instead. This provides a straightforward method to compute the solution. As promised utility in the application of this paper has an important interpretation and the set of feasible promised utility is easy to compute, I use this approach.

The Pareto frontier is defined by

$$
\begin{aligned}
V(s^t, U(s^t)) = \max_{T(s^t),(U(s^{t+1}))_{s^{t+1}}^S} & \ln\left(C^{*U}(s_t)+T(s^t)\right) - \kappa^* l^{*U}(s_t) - v^N(s_t) + \beta \sum_{s^{t+1}}^S p(s^{t+1} \mid s^t) V(s^{t+1}, U(s^{t+1})) \\
\text{s.t.} \quad [\lambda(s^t)] \quad & \ln\left(C^U(s_t)-T(s^t)\right) - \kappa l^U(s_t) - u^N(s_t) + \beta \sum_{s^{t+1}}^S p(s^{t+1} \mid s^t) U(s^{t+1}) \geq U(s^t) \\
[\beta p(s^{t+1} \mid s^t)\phi(s^{t+1})] \quad & U(s^{t+1}) \geq 0 \\
[\beta p(s^{t+1} \mid s^t)\zeta(s^{t+1})] \quad & V(s^{t+1}, U(s^{t+1})) \geq 0 \\
& C(s_t) = C_H^{\gamma}(s_t) C_F^{1-\gamma}(s_t) \\
& Y_H(s_t) = C_H(s_t) + C_H^*(s_t) \\
& Y_F(s_t) = C_F(s_t) + C_F^*(s_t)
\end{aligned}
\tag{19}
$$

The first constraint is the promise keeping constraint for H. The Lagrange multiplier $\lambda(s^t)$ is attached to that constraint. As in Marcet and Marimon (2019), $\lambda(s^t)$ can be interpreted as the planner's weight for H. The next two conditions are the participation constraints, they receive the Lagrange multipliers $\beta p(s^{t+1} \mid s^t)\phi(s^{t+1})$ and $\beta p(s^{t+1} \mid s^t)\zeta(s^{t+1})$ respectively. Notice the timing of the social planner in this setup: The planner chooses a transfer $T(s^t)$ given the overall history and makes a state contingent plan of continuation values for all states in the next period. I show in the Appendix A.12.5 that the Pareto frontier $V_s(\cdot)$ is concave. Therefore, the following first order conditions are necessary and sufficient:

$$
-\frac{\frac{d}{dT(s^t)} u^{*U}(s_t)}{\frac{d}{dT(s^t)} u^U(s_t)} = \frac{C^U(s_t) - T(s^t)}{C^{*U}(s_t) + T(s^t)} = \lambda(s^t) \tag{20}
$$

and

$$
\frac{\lambda(s^t) + \phi(s^{t+1})}{1 + \zeta(s^{t+1})} = -V'(s^{t+1}, U(s^{t+1})) \tag{21}
$$

In addition, the envelope condition is:

$$
\lambda(s^t) = -V'(s^t, U(s^t)) \tag{22}
$$

The optimal contract is therefore characterized by the evolution of $\lambda(s^t)$ over time. $\lambda(s^t)$, according to (22), measures the rate of transformation of the social planner: At which rate can H's surplus be traded ex post against that of F's surplus? The first order conditions also trace out a positively sloped relationship between $U(s^{t+1})$ and actual consumption in H. If promised utility is increased for H, the social planner optimally also increases consumption for the same period[12]. Once the state of nature s^{t+1} in the next period is

[12] This means that if the social planner becomes active and changes the allocation, she uses both policy instruments to increase utility. Current consumption and promised utility

known, the new value of $\lambda(s^{t+1})$ which equals $V(s^{t+1}, U(s^{t+1}))$ is determined by (21). In that case it is important to consider, if the participation constraints bind. As $\lambda(s^t)$ also equals the ratio of marginal utilities of consumption, this pins down the current optimal transfer together with the aggregate resource constraint.

The role of the participation constraints for the allocation of consumption can be illustrated by combining (22) and (21).

$$\frac{-V_s'(U(s^t)) + \phi(s^{t+1})}{1 + \zeta(s^{t+1})} = -V'(s^{t+1}, U(s^{t+1})) \tag{23}$$

There are three regions of interest for state s^{t+1}:

1. Neither participation constraint binds.

No participation constraint binds. This is the case for example when both countries are equally productive. This implies that both Lagrange multipliers are 0 ($\zeta(s^{t+1}) = 0, \phi(s^{t+1}) = 0$):

$$V^{'}(s^t, U(s^{t+1})) = V^{'}(s^{t+1}, U(s^{t+1}))$$

Therefore the country's relative weight for the planner stays the same, $\lambda(s^{t+1}) = \lambda(s^t)$. The intuition is, if no country wants to leave the union, no change in the contract is necessary. The relative weight stays the same, promised utility is unchanged and the ratio of marginal utilities is unchanged as well.

2. F's participation constraint binds.

F wants to leave the union, the participation constraint binds. Therefore $\zeta(s^{t+1}) > 0, \phi(s^{t+1}) = 0$.

$$\frac{-V^{'}(s^t, U(s^t))}{1 + \zeta(s^{t+1})} = -V^{'}(s^{t+1}, U(s^{t+1})) \Rightarrow -V^{'}(s^t, U(s^t)) > -V^{'}(s^{t+1}, U(s^{t+1})) \Rightarrow U(s^t) > U(s^{t+1})$$

Remember that $V'(\cdot) < 0$. If F's participation constraint binds in state s^{t+1}, promised utility $U(s^{t+1})$ for H decreases, compared to the initial promise $U(s^t)$. As a result, H's relative consumption in that period decreases as well. This is done to make F stay in the union, as consumption and expected future utility of F increase to ensure that its participation constraint holds with equality.

3. H's participation constraint binds.

H's participation constraint binds. In that case $\zeta(s^{t+1}) = 0, \phi(s^{t+1}) > 0$ and

$$-V^{'}(s^t, U(s^t)) + \phi(s^{t+1}) = -V^{'}(s^{t+1}, U(s^{t+1})) \Rightarrow -V_s(U(s^t)) < -V(s^{t+1}, U(s^{t+1})) \Rightarrow U(s^t) < U(s^{t+1}).$$

Promised utility and relative consumption is increased in state s^{t+1} to make H stay in the monetary union. H's utility level is given by the binding participation constraint.

As in Ligon et al. (2002), an equation summarizes the dynamics for consumption: There exist state-dependent intervals $[\underline{\lambda}(s^{t+1}), \bar{\lambda}(s^{t+1})]\ \forall s^{t+1} \in S$, such that $\lambda(s^t)$ evolves according to the following rule: Let s^t be given and s^{t+1} be the state at time $t+1$, then

$$\lambda(s^{t+1}) \begin{cases} = \underline{\lambda}(s^{t+1}) & \text{if } \lambda(s^t) < \underline{\lambda}(s^{t+1}) \\ = \lambda(s^t) & \text{if } \lambda(s^t) \in [\underline{\lambda}(s^{t+1}), \bar{\lambda}(s^{t+1})] \\ = \bar{\lambda}(s^{t+1}) & \text{if } \lambda(s^t) > \bar{\lambda}(s^{t+1}). \end{cases} \tag{24}$$

where $\underline{\lambda}(s^{t+1}) \equiv -V'(\underline{U}(s^{t+1}))$ and $\bar{\lambda}(s^{t+1}) \equiv -V'(\bar{U}(s^{t+1}))$ are the endpoints of the equation, indicating whether the participation constraints bind, if the old contract $\lambda(s^t)$ is still in place.

The intuition behind the evolution for $\lambda(s^t)$ is the following: An optimal contract requires that the ratios of marginal utilities of both countries stay constant over time, whenever possible. Transfers are therefore chosen such that the old ratio $\lambda(s^t)$ is the same as the new ratio $\lambda(s^{t+1})$ if all constraints are satisfied. Whenever one of the participation constraints is violated for a certain state and for a given old contract, a new contract is put into place, that engineers the minimum change necessary in marginal utilities to satisfy both participation constraints. That is, put the country that wants to leave the union at its participation constraint by choosing the appropriate transfer. This new contract with its transfer system and its marginal utility ratio is in place as long as possible but will change again when one country is at is participation constraint. In the context of the two-country model, the evolution of $\lambda(s^t)$ has a remarkable feature: As long as no new participation constraint binds transfers in % of GDP are constant over time.[13] This provides a simple rule that helps to sustain the monetary union.

Furthermore, there will be effects on output, employment and prices, as transfers shift consumption from one country to another. These general equilibrium effects are present, because countries have a preference for domestic goods due to their Home bias[14]. For a further discussion of these effects, see section A.10 in the Appendix.

Starting with a certain state s^0, the Pareto frontier [15] can be traced out by letting the initial value $\lambda(s^0)$ vary between the minimum value $\underline{\lambda}(s^0)$ and maximum value $\bar{\lambda}(s^0)$. These contracts correspond to transfers that are chosen in such a way, that the gain of a currency union compared to national currencies is zero or its maximum possible value. A natural starting point are zero transfers with an equal gain split between both countries in the benchmark simulation.

I now outline the algorithm that solves for transfers and the overall allocation in the economy. Given the process for productivity a_t and a_t^*, a history s^t is simulated. Consumption and labor in a currency union and outside a currency union are then computed according to (16) and (14). Starting with zero transfers, the gain (17) is computed and the algorithm checks for which t any of the gains are negative. The algorithm computes

[13]See the Appendix for the proof.

[14]This goes back to an old debate between Keynes and Ohlin in 1929, the so-called Transfer debate. Back then the debate centered around transfers (debt repayments) of Germany to the Allied nations after its defeat in World War I and the general equilibrium effects of these transfers.

[15]See Figure11

the set of feasible promised utilities and in the first period when the participation constraint binds for one country, transfers and the promise for that country are chosen such that the gain is set to zero. The Pareto weight is set to the corresponding endpoint of the state. Future utility (the promise) is explicitly written in state contingent form that include future transfers. These future transfers obey (20) and (24). The condition is, that marginal rate of transformation of the social planner stays the same in all states, except for the other asymmetric state when $\lambda(s^t)$ is inversed. $\lambda(s^t)$ is updated in the period with negative gains. With that, the new ratio of marginal rates of utility is computed that includes transfers, obeying (24), as long as the next country has a positive gain. The promise keeping constraint is checked for all new transfer schemes. The updated lambda is then used, to compute new gains from that moment onward. As soon as another participation constraint binds, the algorithm computes a new $\lambda(s^t)$ as before and updates the allocation.

3.3 Union-wide Central Bank with Exit Option

Now consider the setup as before. Both countries can exit in every period and a social planner maximizes union-wide welfare, taking the lack of commitment of both member states into account. The only difference is that the planner uses monetary policy $\mu(s^t)$ as an instrument instead of transfers. $\mu(s^t)$ summarizes the history of monetary policy until now, if today's state is s. It reflects the path of interest rate that the central bank chooses. The central bank chooses the monetary stance today and promises future utility:[16]

$$
\begin{aligned}
V_s(U(s^t)) = \max_{\mu(s^t),(U(s^{t+1}))^S_{s^{t+1}}} \quad & \ln\left(C^{*U}(\mu(s^t))\right) - \kappa^* l^{*U} - v^N(s_t) + \beta \sum_{s^{t+1}}^{S} p(s^{t+1}\,|\,s^t) V(s^{t+1}, U(s^{t+1})) \\
\text{s.t.} \quad [\lambda(s^t)] \quad & \ln\left(C^{U}(\mu(s^t))\right) - \kappa l^{U} - u^N(s_t) + \beta \sum_{s^{t+1}}^{S} p(s^{t+1}\,|\,s^t) U(s^{t+1}) \geq U(s^t) \\
[\beta p(s^{t+1}\,|\,s^t)\phi(s^{t+1})] \quad & U(s^{t+1}) \geq 0 \\
[\beta p(s^{t+1}\,|\,s^t)\zeta(s^{t+1})] \quad & V(s^{t+1}, U(s^{t+1})) \geq 0 \\
& C(s_t) = C_H^{\gamma}(s_t) C_F^{1-\gamma}(s_t) \\
& l(\mu(s^t)) a(s_t) = C_H(\mu(s^t)) + C_H^*(\mu(s^t)) \\
& l(\mu(s^t))^* a(s_t) = C_F(\mu(s^t)) + C_F^*(\mu(s^t))
\end{aligned} \tag{25}
$$

[16] Monetary policy under commitment implies that only consumption is targeted, but not employment. I adopt the same notion here.

The first order conditions with respect to promised utility $U(s^{t+1})$ are the same, the only difference is the first order condition with respect to the policy instrument $\mu(s^t)$:

$$
-\left[\frac{1}{\mu(s^t)} - \frac{(1-\gamma)a^{-1}(s_t)}{\sum_{s^t}^{S} p(s^t \,|\, s^{t-1})a^{-1}(s_t)\mu(s^t)} - \frac{\gamma a^{*-1}(s_t)}{\sum_{s^t}^{S} p(s^t \,|\, s^{t-1})a^{*-1}(s^t)\mu(s^t)}\right] \cdot
$$

$$
\left[\frac{1}{\mu(s^t)} - \frac{\gamma a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^t \,|\, s^{t-1})a^{-1}(s_t)\mu(s^t)} - \frac{(1-\gamma)a^{*-1}(s_t)}{\sum_{s^t}^{S} p(s^t \,|\, s^{t-1})a^{*-1}(s_t)\mu(s^t)}\right]^{-1} = \lambda(s^t)
$$

Writing in terms of marginal costs of both countries:

$$
1 = \left(\frac{1-\gamma+\lambda(s^t)\gamma}{1+\lambda(s^t)} \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{\gamma+\lambda(s^t)(1-\gamma)}{1+\lambda(s^t)} \frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)^{-1} \tag{26}
$$

The new monetary stance features *time-varying* Pareto weights $\lambda(s^t)$. This means that compared to the initial stance without exit option (15), the central bank stabilizes a time-varying weighted average of marginal costs of both countries. Monetary policy in both regimes is exactly the same if $\lambda(s^t) = \frac{1-\xi}{\xi}$. Hence, the central bank can stick to the structure of the old monetary rule but announce that the effective weights for both countries become state-dependent. If the participation constraint of one country is binding, the central bank puts more weight on stabilizing marginal costs of the crisis country today and promises to do that in the future as well. The mechanism at work is the following. Imagine the Home country is in a severe recession and wants to leave the union. As hours worked are too high, H wants to use national currencies to increase interest rates. A union-wide central bank recognizes that the participation constraint of H binds and increases the effective weight $\lambda(s^t)$ for H. This implies that the central bank favors H when setting interest rates. Union-wide interest rates are higher than they would be without the threat of a break-up. This way the central bank stabilizes marginal costs of the home country more and in turn puts the employment level of home closer to optimum. In addition, the central bank announces to conduct monetary policy in favor of H in the future. The Pareto weight $\lambda(s^t)$ has persistently changed and the gains of H are exactly zero during the crisis period, such that it does not leave the union. $\lambda(s^t)$ stays the same, until another participation constraint will bind.

Are there limitations for the central bank to redistribute resources between countries with interest rate setting? Yes, there is for example no way the central bank can put more weight on one country than on antoher in a symmetric current state of the economy. The best that monetary policy can do in such a situation is to close output and employment gaps of both countries. Only when there are asymmetric states, the central bank can alter the weights to favor a specific country. Therefore, the ability to make credible promises about future utility is limited for the central bank. The paper considers an example, in which transfers can sustain the union, while interest rate setting alone cannot.

3.4 Union-wide Central Bank and Transfers with Exit Option

Here I consider a joint response of both, monetary policy and fiscal policy. In the period in which the participation constraint of one country is binding, given the policy in place from the past, the central bank re-calibrates the weight of the country only for this period. In a next step, the fiscal authority, taking the new monetary policy today into account, sets fiscal transfers as in section 3.2 and tries to sustain the union. I will consider two possibilities for the central bank. The first features an increase of the weight for the crisis country to one. The second option includes a drop in the weight for the crisis country to zero, this coincides with an increased economic activity for the whole union. In the experiment I will check if any of these two options increases the survival rate of the currency union, compared to other policy interventions.

4 Calibration

The section calibrates the model. The model seeks to highlight conditions under which a currency union such as the eurozone can break up. Towards that aim, I focus on two large members of the eurozone, namely Germany and Italy. The choice of Italy and Germany as our countries of interest has a reason: Both are the largest countries of their respective block: Germany being part of the so-called core (or the northern) block in the currency union, where the economy in the last twenty years expanded significantly. And Italy as the largest country of the so-called periphery (or the southern block) that experienced large economic downturns. I will use data from these two countries to calibrate trade openness and real interest rates. One period in the model taken to be a year.

Other parameters are calibrated based on the outside literature. Furthermore, a range of trade costs parameters will be considered, implying different amounts of gains coming from the union.

4.1 Calibration of Preferences and Technology

Both Home and Foreign are assumed to be symmetric in their parameters. The discount factor β of the representative household is set to 0.98 to match a yearly real interest rate of about 2 % in line with Brand et al. (2018) for the eurozone. The Home bias parameter γ is set to 0.75 which is in line with Italy's trade openness in 2015 measured as imports relative to GDP[17]. The elasticity of substitution between domestic goods is set to 6 as in Galí (2008) implying a markup of 20%. I have made the following implicit assumptions by choosing preferences as in equation (1). The intertemporal consumption elasticity is set to 1 so that consumption utility is log. As in Corsetti and Pesenti (2002), labor is just linear implying an infinite Frisch elasticity of labor supply, such that household satisfy labor demand. κ is set to 8/3 so that household spend one third of their time with labor.

[17] According to Eurostat imports relative to GDP in 2015 for Italy is 26.7%.

The trade elasticity of substitution between Home and foreign goods is set to 1, so that the consumption aggregator in (2) is Cobb Douglas. Together with the assumption of log consumption, this implies that the current account is always balanced which is numerically convenient, see also section A.1. A trade elasticity of 1 is at the lower end of available estimates surveyed by Head and Mayer (2014). Estimates vary widely. Lower values of the trade elasticities are in most cases related to measurements of short-run elasticities. Low values of trade elasticity imply for the model, that a reduction in trade costs has a smaller effect on the trade volume[18].

Table 1: Calibration

Symbol	Value	Description	Target
β	0.98	Time discount rate	Real rate of 2% p.a.
γ	0.75	Home bias for each country	Trade openness Italy 2015
$\theta, (\theta^*)$	6	Elasticity of substitution of Home goods	Galí (2008)

Next, I discuss choices for parameters that are central for the motives of forming a currency union. They are summarized in table 2. The calibration of ϖ is crucial, as it determines the trade gains from a currency union. If gains are very large, a currency union would always be formed and would never break up. Instead, if the gains are low, the monetary union becomes fragile. For example, Micco et al. (2003) find that bilateral trade increases by around 4-16% if a common currency is adopted. This is a bit higher than estimates by Baldwin et al. (2008) (5%), but much lower than Rose (2000). Therefore, the paper considers several specifications with large, medium and small trade gains that are in line with the wide range of estimates that the literature finds.

Table 2: Calibration of union trade gains

Symbol	Large gains	Medium gains	Small gains	Description
ϖ	0.1	0.066	0.05	Transportation costs.
ξ	0.5	0.5	0.5	Weight of H
τ	-0.2	-0.2	-0.2	Subsidy, no markup.

As gains come from trade costs reduction only, benefits of a currency union are symmetric between both countries. In the "large gains" scenario I set ϖ in such a way that with national currencies 10% of all exported goods are lost. The elimination of trade costs generates an increase of bilateral trade of 10 % in good times. Taken the productivity process into account, the currency union would never break up. Gains are so large that no country would voluntarily leave the union

[18] As discussed below, I calibrate the trade costs in such a way, that bilateral trade increases between 3% and 10 %.

Consider the "medium gains" calibration. Given the same productivity process, trade costs reduction ϖ is 6.5%. In this specification with lower trade gains, the currency union can actually break up when the biggest possible asymmetric shock emerges, see section 5. Last I will discuss low trade gains of 5% in line with estimates from Baldwin et al. (2008). The union is more likely to break up in that specification, as governments decide to leave the union also in those states in which relatively small asymmetric shocks occur.

5 Model Experiment and Results

I want to capture, how each of the planners in section 3 fares when productivity fluctuates stochastically over time. For this purpose I run a simulation of the model and compare the outcome of each planner in the simulation. This table reminds of the policy instruments used by each planner

Table 3: Planners and which policy instruments is used to prevent a breakup

Planner Allocation	Transfers are used	Interest rates are used
National Planner	-	-
Union-wide Ramsey Planner	✓	-
Union-wide Central Bank	-	✓
Transfers & Mon Pol	✓	✓

Consider the model with overall 25 possible states, in which each country can have 5 different productivity values: $\mathcal{A} = \{a^{bb}, a^b, a^n, a^r, a^{rr}\}$, where $a^{bb} = 1.04$ indicates a big boom with very high productivity. It indicates GDP growth of 4%. $a^b = 1.02$ is a normal boom with higher productivity, $a^n = 1$ a neutral state and a^r, a^{rr} indicate recessions of equal size as the bomm. In that setup, there are 5 symmetric states, in which both countries have the same productivity, 20 are asymmetric. I assume that productivity is independent between countries and over time. The probability for each country to have productivity $\mathcal{A}$ is $Prob = \{0.15, 0.4, 0.25, 0.15, 0.05\}$. The first entry corresponds to the probability to go get productivity $a^{bb} = 1.04$. Each simulation has 100 periods. Overall, I run 2500 simulations. In a next step, I use the baseline calibration discussed in section 4 for the simulation. In addition to that, I use different amount of gains from trade simulations.

5.1 Trade Gains 6.5%

The simulation is used to compute average consumption and employment with trade gains of 6.5%. This is done by considering the pure allocations of a national currency (14), a currency union (16) and the first best allocation (A.1). No planner intervention is considered yet.

Table 4: Allocation under different regimes, trade costs reduction of 6.5%

Planner Allocation	Average Consumption	Average labor
National Currency	14.144	24.61
Currency Union	14.3703 (+1.6%)	25 (+1.57%)
First Best	14.3723 (+1.61%)	25 (+1.57%)

With trade costs reduced by 6.5%, consumption of both countries increases by around 1.6% in a currency union. I take one simulation out as an example. Consider how productivity evolves over time, starting from a point in which both countries are in a boom:

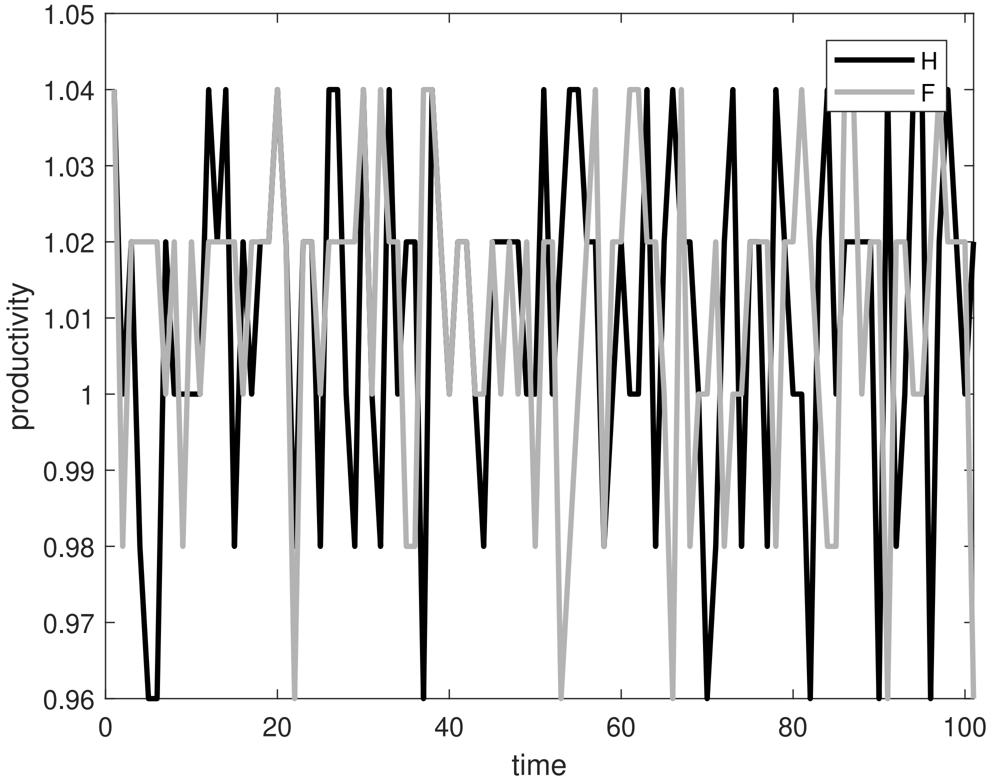

Figure 1: Productivity of both countries over time.

Given the evolution of productivity, I compute consumption and employment over time. The gains from the currency union in (18) are then computed. This allows us to check if the participation constraint of the union holds in this specific simulation.

National Planner:

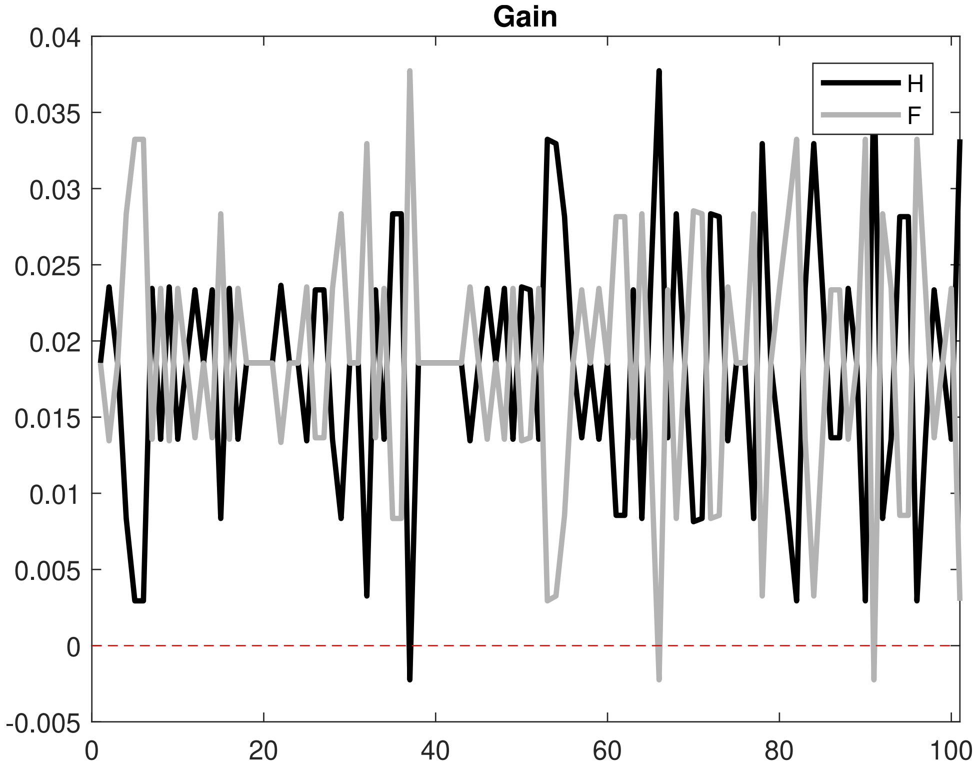

Figure 2: Gain of both countries over time.

The values for H and F fluctuate around 0.2. Both countries' gains are exactly identical when productivity is the same. If there are asymmetric shocks, the recession country's gain goes down while the boom country's gain goes up. In that example, only the biggest possible asymmetric state can endanger the currency union. For trade gains larger than 6.7% gains are always positive and the union would never collapse. For the specification in this simulation, gains turn negative in period 36. At that point in time, there is a huge asymmetric shock with the Home country being in a deep recession, while the Foreign country is in a big boom. The gain of the Home country is negative and the government of that country wants to leave the currency union. The next point in time, when gains turn negative is in period 65. Then, the Foreign country's gain is negative and its government wants to leave the union. As discussed in 3.1, the union breaks up as soon as the first gain turns negative. Both countries have zero gain from that moment onward.

Transfers: A union-wide Ramsey planner with transfers between countries, as in section 3.2 sets transfers in the following way to prevent that break-up:

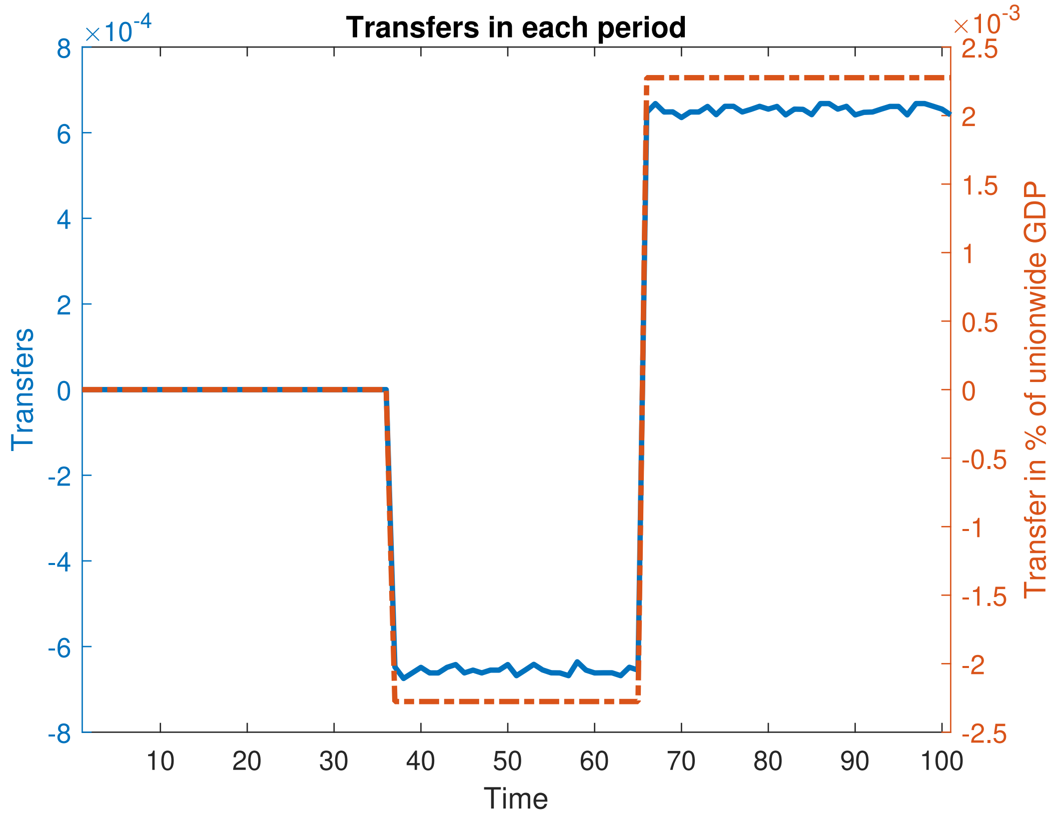

Figure 3: Transfers bz the Ramsey planner over time. The solid blue solid line are real transfers in terms of consumption units (scale on the left axis), while the red dashed line are transfers in percent of union-wide GDP (scale on the right axis).

When the huge asymmetric shock emerges in period 36, the Ramsey planner gives transfers to the recession country H, T_t is negative. Furthermore she makes a promise that a constant fraction of union-wide GDP is redistributed to the Home country in every period, until another participation constraint binds again. This happens in period 66, as the Foreign country enters a severe recession and the Home country experience a strong boom. Transfers turn positive in that period to prevent the Foreign country to exit. The transfer scheme reverses. In this example, a transfer of 0.0024% of union-wide GDP every period sustains the union. As in Ferrari et al. (2020), the relative amount stays constant whenever possible, reflecting a persistent increase in the Pareto-weight. As there are also aggregate fluctuations in my model, the absolute amount of transfers (solid blue line) varies over time together with the economy.

Central Bank: If fiscal transfers between countries are not feasible, can a union-wide central bank with interest rate setting, as in section 3.3 prevent a break-up? In this example, the answer is yes. In period 36, the central bank alters its monetary stance to favor the Home country. The central bank does not only favor the Home country in the crisis period, but also in the future. The following figure illustrates the new behavior of interest rates around the first crisis in period 36.

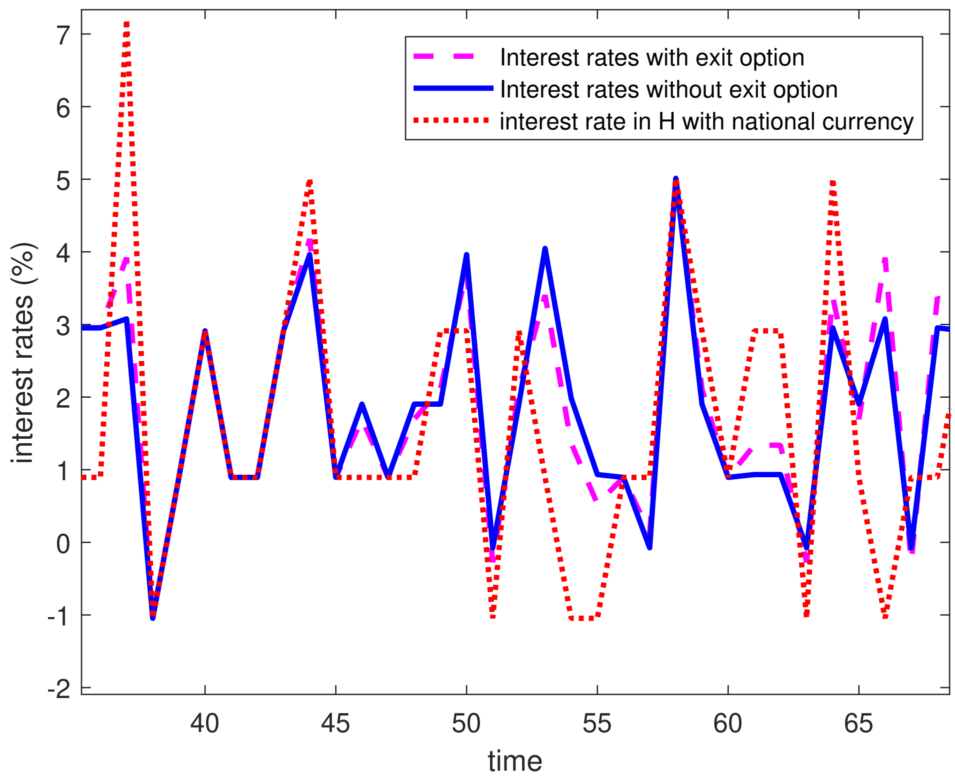

Figure 4: Interest rates over time in different regimes.

The new interest rate (dashed magenta line) is closer to what interest rates would be for H with national currencies (the dotted red line). Remember that in a currency union without exit options (the solid blue line) both countries have an equal weight in the central bank's objective function. When there are exit options, the weight in the objective function becomes time-varying and state-dependent to take care of the participation constraints. The big asymmetric shock makes H's participation constraint binding, which leads to a persistently higher weight of H in the central bank's objective function. By increasing H's weight, the central bank makes sure that H stays in the currency union. The higher weight of H persists until F's participation constraint binds. Therefore, in the crisis period 36 interest rates are closer to what H wants with national currencies: With very low productivity in H, the central bank increases interest rates to lower aggregate demand, which is in H's favor. In period 36, interest rates are at 4% rather than at 3% as they would normally be. In period 37 to 42 all interest rates align, as productivity is the same in both countries. This means that the union-wide central bank has no room to set interest rates in one country's favor, as both want exactly the same interest rate. In other asymmetric states after the big shock in 36, the central bank systematically sets interest rates in favor of the Home country. The Pareto weight changes in period 66, when F hits its participation constraint. From that moment on, the central bank favors F in its policy stance. This example highlights the conditions necessary for the central bank to succeed to sustain the union: There have to be sufficiently many 'small' asymmetric shocks, that the central bank can use to favor a country without endangering the union. If there are no such states with small asymmetric shocks, the central bank cannot credibly promise to give the crisis country more utility in the future. In this example, there are sufficiently many small asymmetric shocks that are also likely to occur.

The following graph illustrates this point. I plot interest rates in a union over time in this simulation, together with the set of all possible interest rates that would favor one or another country. The set of possible interest rates is computed by considering all possible weights $\xi \in [0, 1]$ in the central bank's monetary stance (15). In the extreme, the central bank puts full weight on H or F respectively. The weights are reflected in consumption (16) and with the Euler equation (6) the set of all possible interest rates are computed.

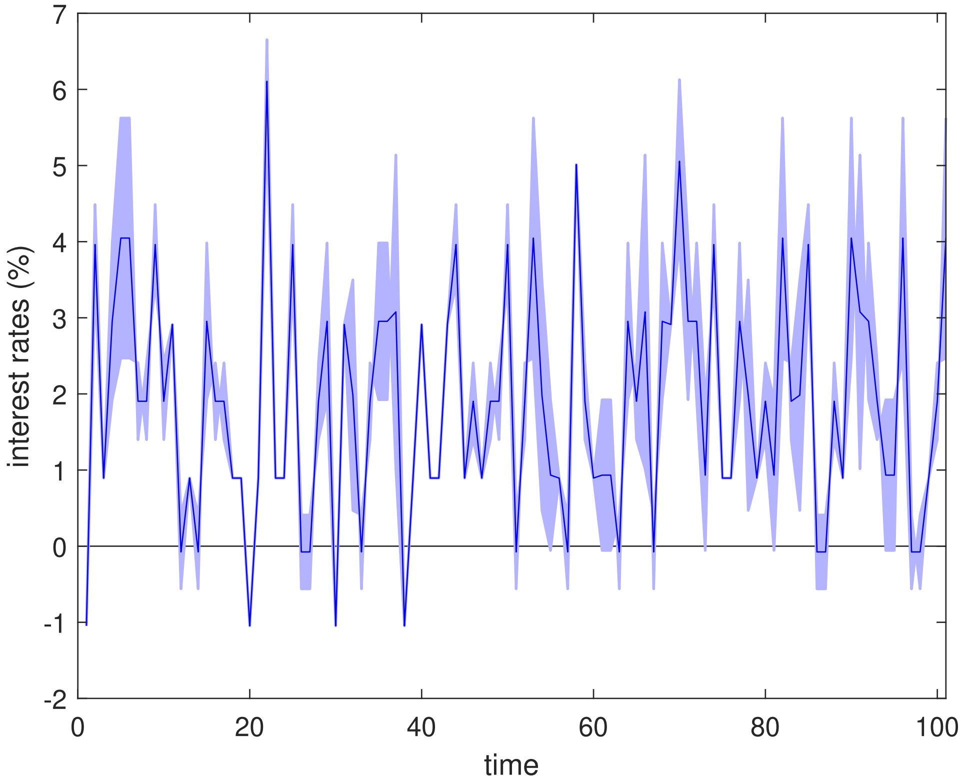

Figure 5: Interest rates over time. The shaded area illustrates the range of interest rates that a union-wide central bank can optimally implement, when putting full weight on H or F respectively. The solid blue line indicates interest rates with equal weight.

There are periods, in which the set of possible interest rates is just one point. In these periods, productivity is the same for both countries, implying that both want the same interest rates. This leaves no room for the central bank to favor a country.

Central Bank and Transfers: In the three periods 36, 66 and 91 with huge asymmetric shocks the central bank puts full weight on the countries that want to leave the union. Emphasizing stabilization of crisis countries during an asymmetric shock alters interest rates in those periods and reduces the amount of necessary transfers to sustain the union from 0.0025% to 0.0017%, see figures 18 and 19.

Summary: Turn to the statistics that describe the likelihood of a break-up for different planner intervention. Given the productivity process in the simulation, the currency union experiences in 1.5 % of the time a huge asymmetric shock that endangers the union. In 79.8% of all 2500 simulations, such a shock actually occurs within the first 100 periods and the currency union breaks up if national planners are in charge. The average break-up period is 81.8. The last column summarizes average gains in this simulation. With a national planner, the average gain is lowest as the currency union breaks up relatively often in the simulation. All other planners are able to increase the average gain substantially, as they succeed to sustain the union and the trade gains. Monetary policy fairs slightly worse than other interventions, as interest rates are a distortionary policy instrument. The following table summarizes these results.

Table 5: Break-up under different planners, trade costs 6.5%

Planner Allocation	Prob. of a union break-up next period	Average break-up period	Prob. of a break-up within 100 periods	Average Gain
National	1.5 %	81.8	79.8%	0.150
Fiscal	0 %	-	0%	0.189
Monetary	0 %	-	0%	0.188
Fiscal & Mon	0 %	-	0%	0.189
First Best	0 %	-	0%	0.189

Overall, a union-wide transfer scheme always succeeds to sustain the currency union, as does a common central bank.
In a next step, I consider lower trade gains coming from a currency union.

5.2 Trade Gains 5%

This section discusses how lower trade gains affect the effectiveness of policy instruments that aim to sustain the union. First I consider a specific example with lower trade gains, then I show for which ranges of gains in a currency which policy works. The following table summarizes the effect of a trade costs reduction of 5%:

Table 6: Allocation under different regimes, trade costs reduction of 5%

Regime	Average Consumption	Average labor
National Currency	14.1957	24.6984
Currency Union	14.3705 (+1.23%)	25 (+1.22%)
First Best	14.3725 (+1.24%)	25 (+1.22%)

A 5 % decrease in trade costs in a currency union increases consumption in the simulation by around 1.23 %, employment by around 1.22%. The starting point of the simulation is again a strong boom for both countries (a^{bb}). Consider a random simulation that I have taken out as an example. As before I consider first the outcome of the experiment of the national planner, then the Ramsey planner with transfers, then the union-wide central bank and then the joint intervention.

National Planner First I plot the evolution of gains, as in (18) to check in which point in time a national planner decides to leave the currency union. As productivity diverges, so do gains. The Foreign country experiences a recession and its gains from the currency union go down. They turn negative in period 8, when an asymmetric shock emerges. Afterwards, each countries' gains get closer to each other, as productivity of both countries aligns again. The union would collapse in period 8 and both countries receive zero gains from that moment onward.

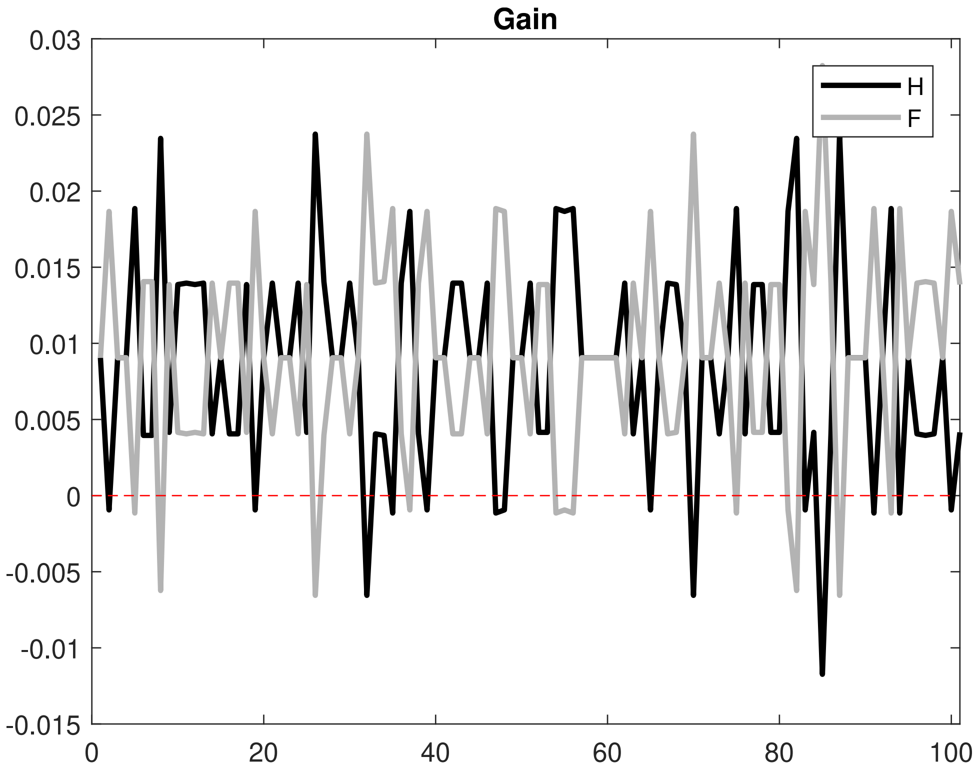

Figure 6: Gains over time .

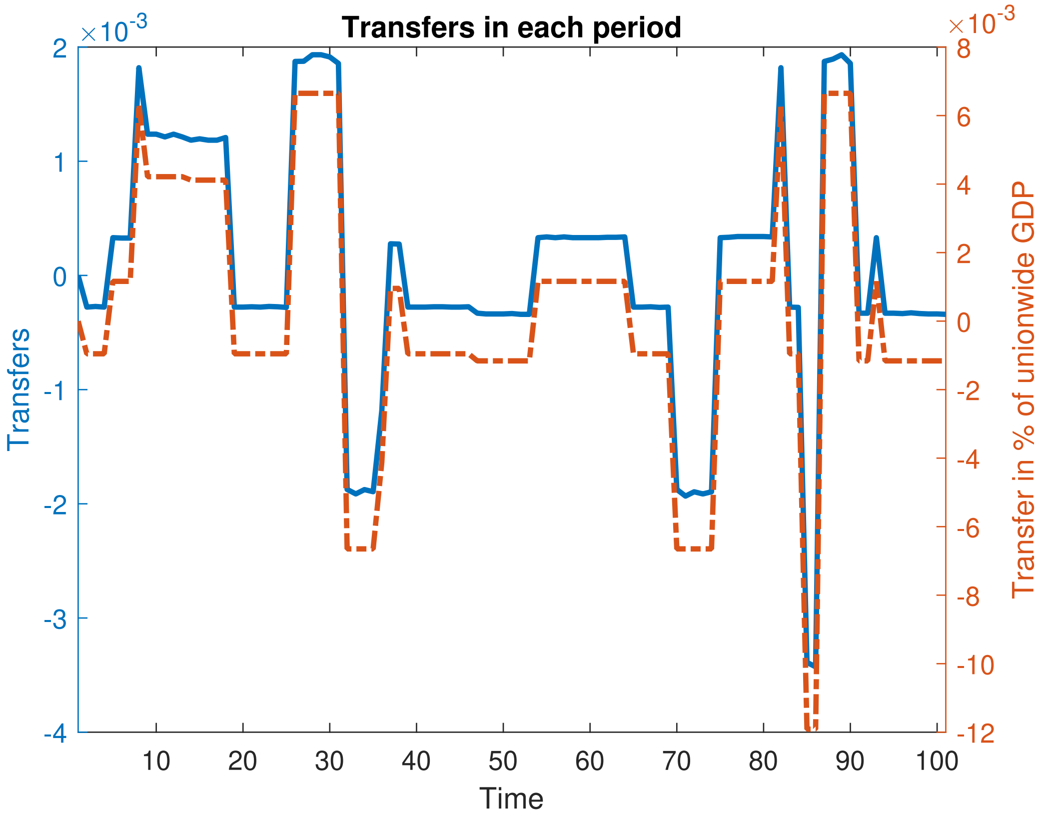

Figure 7: Transfers over time. The solid blue solid line are real transfers in terms of consumption units (scale on the left axis), while the red dashed line are transfers in percent of union-wide GDP (scale on the right axis).

Transfers A union-wide social planner with transfers, as in 3.2 sets transfers as in Figure 7. Compared to figure 3, transfers fluctuate stronger as the participation constraints of both countries are hit more frequently. In addition to that, transfers have to be changed as well if one country would leave the union, because the current transfer scheme puts it into a disadvantage. This is true for example in period 15. Still transfers can always sustain the union by ensuring that gains are not negative.

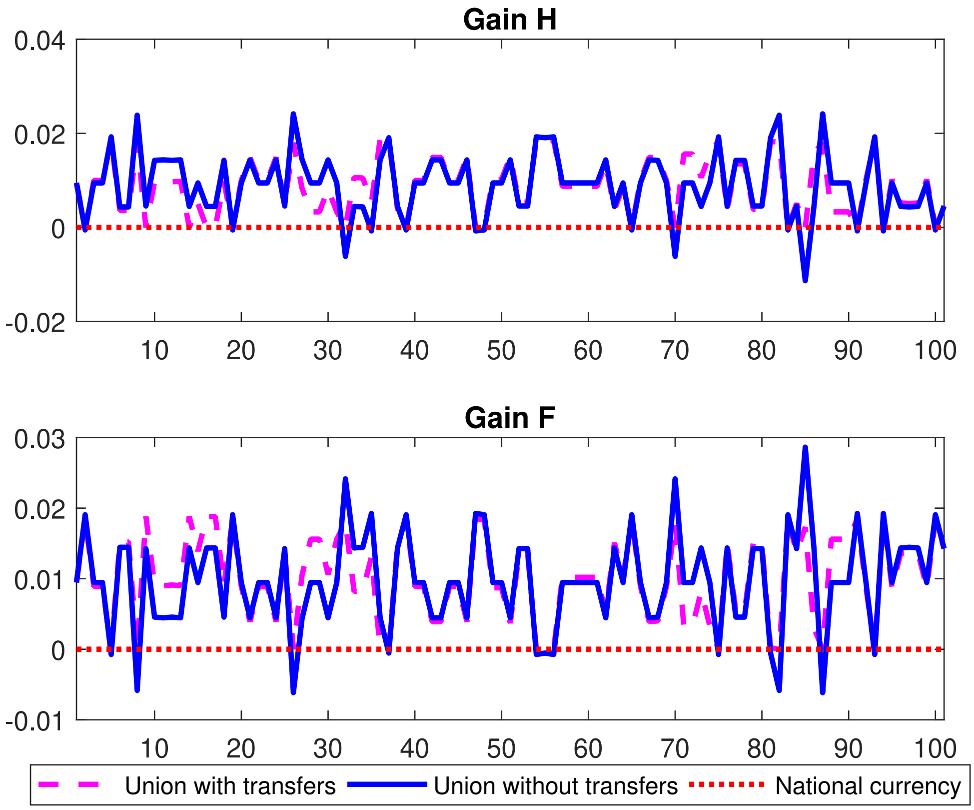

Figure 8: Gains over time under different policy regimes.

Central Bank: Can the central bank in this simulation sustain the union? Only for some time. The following graphs zoom into the first 10 periods of the simulation and illustrate this point. The first two asymmetric shocks that would destroy the union under national planners, can be addressed with interest rate setting by the central bank. First in period 2, there is an asymmetric shock that makes H want to leave the Union and in period 5, in which F wants to leave the Union. In both cases, the central bank steps in by accommodating the corresponding crisis country during the crisis period and afterwards. In period 8 however, F is hit again by an asymmetric shock, but this time the shock is so large that the central bank cannot sustain the currency union, even if she puts full weight on F. Despite the central bank's best effort to keep F in the union, the gain is still negative and the government decides to leave the union. In this simulation, the central bank is able to extend the survival of the currency union for 6 periods, but not to permanently sustain it.

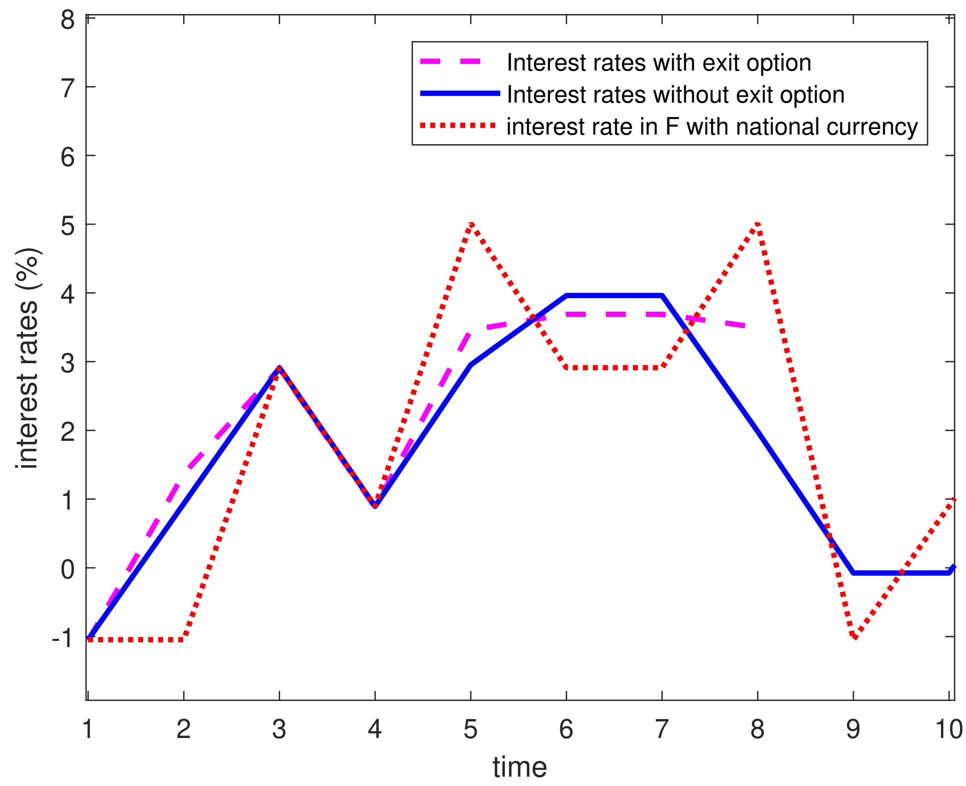

Figure 9: Interest rates over time under different policy regimes, trade gains are 5%.

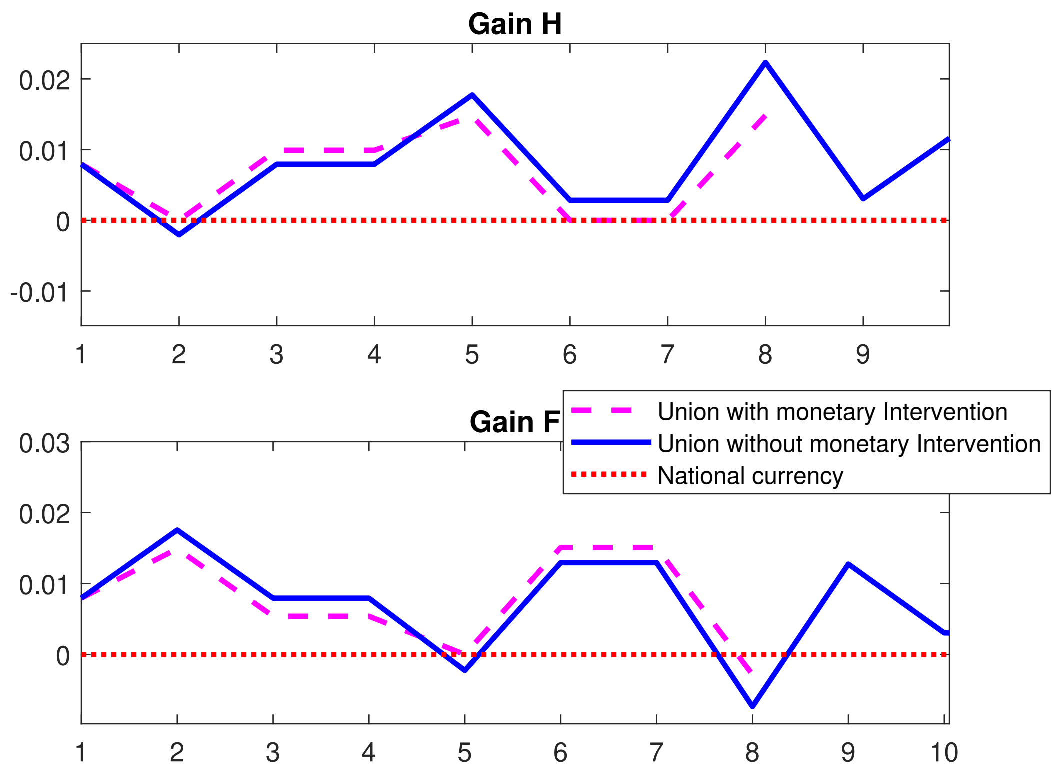

Figure 10: Gains over time under different policy regimes, trade gains are 5%.

Transfers and Central Bank: A central bank that puts a full focus on stabilizing crisis country in the crisis periods, reduces the amount of transfers necessary only by a very small margin, see figure 23.

Summary: Overall, a union-wide transfer scheme always succeeds to sustain the currency union in the benchmark simulation, while a common central bank fails to achieve that. What a central bank can do is to address the threat of a break-up in some states. This reduces the probability of a shock that destroys the currency union in the next period from 32 % to 10%. With a common central bank that tries to prevent a break-up the average duration of a currency union is increased from 2.7 years to 8.4 years.

Table 7: Break-up under different planners, trade costs 5%

Planner Allocation	Prob. of a union break-up next period	Average break-up period	Prob. of a break-up within 100 periods	Average Gain
National	32 %	2.7	100%	0.0002
Fiscal	0 %	-	0%	0.0076
Monetary	10 %	8.4	100%	0.0007
Fiscal & Mon	0 %	-	0%	0.0076
First Best	0 %	-	0%	0.0091

The next table summarizes the policy options that manage to sustain the union, depending on how large trade gains are.

Table 8: Break-up under trade gains

Trade Gains	Probability of dangerous shocks	Transfers can always sustain the union	Central bank can sustain the union
$> 6.7\%$	0%	yes	yes
$[6.6\%, 6.4\%]$	1.5 %	yes	yes
$[6.3\%, 3.3\%]$	$[1.5\%, 73\%]$	yes	no
$3.3\% >$	$73\% >$	no	no

If trade gains are larger than 6.7% no country would ever decide to leave the union, no policy interventions are necessary and therefore the union is sustained forever. For trade gains between 6.6% and 6.4% there is a possibility that the union breaks up if the biggest possible asymmetric shock hits the union. Both, fiscal and monetary policy succeed in sustaining the union. If trade gains are lower than 6.4% monetary policy will not always sustain the union, as the costs of stabilization in the union are too large when a big asymmetric shock hits the union. The gains of the union cannot be sufficiently redistributed with interest rate setting alone. Transfers however always manage to sustain the union, up to trade gains to 3.3%. If trade gains are lower than this, even transfers between countries cannot sustain the union. A joint fiscal and monetary intervention does not increase the survival rate of a currency union, independent if the central bank puts full weight on crisis countries in the crisis period or induces an economic boom in the currency union to increase the available amount of fiscal transfers between countries.

6 Conclusion

This paper shows how a currency union can be sustained with fiscal and monetary policies when member states have an exit option. If there is a big asymmetric shock, trade gains in a union are outweighed by less effective monetary policy. The recession country is severely affected, as gaps in the level of employment are more hurtful in a recession than in a boom. Therefore, the recession country exits in a severe crisis and the union collapses. The paper discusses, how the currency union can be sustained via fiscal or monetary policies. The first option is a fiscal intervention by a union-wide Ramsey planner: A simple and credible transfer rule gives the crisis country a constant fraction of union-wide GDP over time. This is enough to prevent a breakup of the union. These transfers are in place as long as the other country of the currency union is not in a crisis. If a crisis happens and the other country wants to exit, the rule is reversed and the new crisis country gets transfers. In the benchmark simulation of the model, the currency union can always be sustained with transfers. Both countries are better off ex post and ex ante compared to a situation when no policies are in place that sustain the currency union. The second option that the paper considers is monetary policy. If there are no fiscal transfers, the central bank can take the lack of commitment from the countries into account. In normal times, the central bank stabilizes a weighted average of the economy from both countries. The weights depend on the size of the economy and on the Pareto weights for the country. This paper derives that these weights become state-dependent when there are participation constraints: As soon as one country hits the participation constraint, the weight of that country increases and the central bank systematically favors the crisis country in its policy. The greater weight persists, until another participation constraint binds. In some situations, the central bank can sustain the union with that policy, but not in all. The central bank needs sufficiently many small asymmetric shocks in the future that can be used to favor a specific country. In addition to that, large trade gains from the currency union are needed. If this is not the case, the central bank has not enough room to favor one country and fails to sustain the union. A joint intervention of the union-wide central bank and fiscal transfers does not increase the survival rate of a currency union compared to a situation when only transfers are used.

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A Appendix

A.1 The Current Account and Risk Sharing

As in the model of Corsetti and Pesenti (2001) the current account is balanced in all points in time, if the model is initialized with zero wealth. This goes back to work by Cole and Obstfeld (1991) and is reflected in the model: Consumption risk is shared efficiently, there is no need for debts or savings in certain states. Intuitively, risk sharing is ensured via endogenous terms of trade movements. The terms of trade are defined as the relative price of domestic imports in terms of domestic exports, in case of PCP, as in (5) $\mathcal{T} = \mathcal{E}P_F^*/P_H$ Consider a productivity boom in H. In such a situation an expansionary policy is optimal for the central bank in H. Therefore, the exchange rate depreciates, terms of trade depreciate as well. With one unit of F's currency, more units of H's currency can now be bought. Productivity has increased the production of h-type goods, therefore the nominal value of H's exports measured in its currency has increased. At the same time, it has become more expensive for H to buy non-domestic goods. In this special setup[19] the nominal value of exports always equals the nominal value of imports due to that mechanism. Note that F has to pay less for h-type goods in terms of F's currency due to H's exchange rate depreciation. Therefore, even though F does not produce more goods, it can afford to buy more h-type goods without running a current account deficit. With this mechanism in place H's productivity increase spills over to the other country.
In a currency union the exchange rate is fixed and terms of trade movements cannot absorb any asymmetric shocks hitting the economy. Another mechanism of the model makes sure that in such a situation the current account is balanced: As the central bank stabilizes the average of the economy, wedges in the labor market occur. For the boom country monetary policy is not expansionary enough creating a negative wedge, while for the recession country it is too expansionary creating a positive wedge. As a result, employment in the recession country is higher and in the boom country it is lower. With the special setup considered in this paper, overall production of both countries in the currency union is the same. Current accounts are therefore also balanced with asymmetric shocks.

A.2 International Relative Prices

Balanced Current Accounts

The current account is balanced all the time for both monetary regimes, value of imports

[19] The elasticity of substitution between Home and foreign goods is 1, as is the intertemporal elasticity. Furthermore, firms use producer currency pricing.

equal values of exports:

$$P_{F,t}C_{F,t} = \mathcal{E}_t P^*_{H,t} C^*_{H,t}$$
$$\Phi^*(1+\varpi)\mathcal{E}_t \mathbb{E}_{t-1}[MC^*_t]\frac{(1-\gamma)a^*_t}{\Phi^*(1+\varpi)\kappa^*} = \Phi\mathcal{E}_t(1+\varpi)\frac{1}{\mathcal{E}_t}\mathbb{E}_{t-1}[MC_t]\frac{(1-\gamma)a_t}{\Phi(1+\varpi)\kappa}$$

Trade costs cancel each other out, they do not matter for a balanced current account. Plugging in marginal costs and the equilibrium exchange rate gives

$$\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}} / \frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu^*_t]}{a_t^{*-1}}\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu^*_t]\frac{(1-\gamma)a^*_t}{\kappa^*} = \mathbb{E}_{t-1}[a_t^{-1}\mu_t\kappa]\frac{(1-\gamma)a_t}{\kappa}$$
$$\Big(\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu^*_t]}{a_t^{*-1}}\Big)^{-1}\mathbb{E}_{t-1}[a_t^{*-1}\mu^*_t]a^*_t = \Big(\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}\Big)^{-1}\mathbb{E}_{t-1}[a_t^{-1}\mu_t]a_t$$

which is true. Intuitively, in a world with producer currency pricing and elasticity of substitution of 1 between Home and foreign goods, terms of trade movements make sure that risk is perfectly pooled in that economy. This means that the current account between both countries is balanced all the time.

Consumption Risk Sharing

Each country consumes a constant fraction of the produced good in all regimes, as given by the analytic expression for consumption.

A.3 Derivations

A.3.1 Allocation of the Social Planner

An interesting benchmark allocation for the model of the economy is the allocation of the social planner. I assume that the social planner can freely allocate labor and consumption and faces no trade costs. She maximizes welfare of all agents subject to the resource constraints of the economy:

$$\max_{\{C_t, L_t, C^*_t, L^*_t\}} \quad \mathbb{E}_t\Bigg[\sum_{\tau=t}^{\infty}\beta^\tau\Bigg(\ln(C_\tau) - \kappa L_\tau\Bigg)\Bigg] + \mathbb{E}_t\Bigg[\sum_{\tau=t}^{\infty}\beta^\tau\Bigg(\ln(C^*_\tau) - \kappa^* L^*_\tau\Bigg)\Bigg]$$
$$s.t. \quad Y_t(h) = L_t(h)a_t = \underbrace{\int_0^1 C_t(h,j)dj}_{C_t(h)} + \underbrace{\int_0^1 C^*_t(h,j^*)dj^*}_{C^*_t(h)}$$
$$Y_t(f) = L_t(f)a_t = \underbrace{\int_0^1 C_t(f,j)dj}_{C_t(f)} + \underbrace{\int_0^1 C^*_t(f,j^*)dj^*}_{C^*_t(f)}$$

This problem can be written as:

$$
\begin{aligned}
\max_{\{C_t, L_t, C_t^*, L_t^*\}} \quad & \mathbb{E}_t\Bigg[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau)-\kappa L_\tau\Big)\Bigg]+\mathbb{E}_t\Bigg[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau^*)-\kappa^* L_\tau^*\Big)\Bigg] \\
& s.t. \quad Y_t = L_t a_t = C_{H,t} + C_{H,t}^* \\
& \qquad\quad Y_t^* = L_t^* a_t^* = C_{F,t} + C_{F,t}^* \\
& \qquad\quad C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} \\
& \qquad\quad C_t^* = C_{H,t}^{*1-\gamma} C_{F,t}^{*\gamma}
\end{aligned}
$$

She then determines the optimal amount of labor, which produces the goods given the technological constraints and then allocates the goods to each consumer. The Lagrangian is given by:

$$
\begin{aligned}
\max_{C_{H,t}, C_{F,t}, C_{H,t}^*, C_{F,t}^*, L_t, L_t^*} L =& \gamma\ln(C_{H,t}) + (1-\gamma)\ln(C_{F,t}) - \kappa L_t + (1-\gamma)\ln(C_{H,t}^*) + \gamma\ln(C_{F,t}^*) - \kappa^* L_t^* \\
&+ \lambda_{1t}(a_t L_t - C_{H,t} - C_{H,t}^*) + \lambda_{2t}(a_t^* L_t^* - C_{F,t} - C_{F,t}^*)
\end{aligned}
$$

The first order conditions are:

$$
\begin{aligned}
L_{L_t}: &\quad \kappa = \lambda_{1t} a_t & L_{L_t^*}: &\quad \kappa^* = \lambda_{2t} a_t^* \\
L_{C_{H,t}}: &\quad \frac{\gamma}{C_{H,t}} = \lambda_{1t} & L_{C_{F,t}}: &\quad \frac{1-\gamma}{C_{F,t}} = \lambda_{2t} \\
L_{C_{H,t}^*}: &\quad \frac{1-\gamma}{C_{H,t}^*} = \lambda_{1t} & L_{C_{F,t}^*}: &\quad \frac{\gamma}{C_{F,t}^*} = \lambda_{2t}
\end{aligned}
$$

Combining these conditions, the allocation of the social planner is:

$$
C_{H,t} = \frac{\gamma}{\kappa} a_t \qquad C_{H,t}^* = \frac{1-\gamma}{\kappa} a_t \tag{A.1}
$$

$$
C_{F,t} = \frac{1-\gamma}{\kappa^*} a_t^* \qquad C_{F,t}^* = \frac{\gamma}{\kappa^*} a_t^* \tag{A.2}
$$

$$
L_t = \frac{1}{\kappa} \qquad L_t^* = \frac{1}{\kappa^*} \tag{A.3}
$$

A.3.2 Market Economy: Consumer's Problem

In the market economy, each individual maximizes her own utility. the Lagrangian of that maximization problem is given by:

$$\begin{aligned} L(h=j) =& \mathbb{E}_t\Big[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau) - \kappa L_\tau \\ & + \lambda_\tau\big(- B_{H,\tau} + (1+i_{\tau-1})B_{H,\tau-1} - \mathcal{E}B_{F,\tau} \\ & + (1+i^*_{\tau-1})\mathcal{E}_\tau B_{F,\tau-1} + \int \Pi_{t-1}(h)dh - P_{H,\tau}C_{H,\tau} - P_{F,\tau}C_{F,\tau} + W_\tau L_\tau\big)\Big)\Big] \end{aligned}$$

Consumption C_t consists of a combination of a Home and foreign consumption bundle given by:

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma}$$

We can obtain the first order conditions (focs) with respect to $C_{H,\tau}, C_{F,\tau}, L_\tau, B_{H,\tau}, B_{F,\tau}$:

$$\begin{aligned} L_{C_{H,t}}: & \quad \frac{\gamma}{C_{H,t}} & =& \lambda_t P_{H,t} \\ L_{C_{F,t}}: & \quad \frac{1-\gamma}{C_{F,t}} & =& \lambda_t P_{F,t} \\ L_{L_t}: & \quad \kappa & =& \lambda_t W_t \\ L_{B_{H,t}}: & \quad \lambda_t & =& \beta\mathbb{E}_t[\lambda_{t+1}(1+i_t)] \\ L_{B_{F,t}}: & \quad \mathcal{E}_t\lambda_t & =& \beta\mathbb{E}_t[\mathcal{E}_{t+1}\lambda_{t+1}(1+i^*_t)] \end{aligned}$$

and the budget constraint

$$\begin{aligned} & B_{H,t} + \mathcal{E}_t B_{F,t} \leq (1+i_{t-1})B_{H,t-1} - T_t + W_t L_t \\ & + (1+i^*_{t-1})\mathcal{E}_t B_{F,t-1} + \int_0^1 \Pi_{t-1}(h)dh - \int_0^1 p_t(h)C_t(h,j)dh - \int_0^1 p_t(f)C_t(f,j)df \end{aligned}$$

Using the first two focs and taking a geometric average with weights γ and $1-\gamma$ gives:

$$\gamma^{\gamma}(1-\gamma)^{1-\gamma} = \lambda_t (P_{H,t}C_{H,t})^{\gamma}(P_{F,t}C_{F,t})^{1-\gamma}$$

which yields

$$\lambda_t = \frac{1}{P_t C_t}$$

where

$$P_t \equiv \frac{P_{H,t}^{\gamma} P_{F,t}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

is defined as the utility-based price index in country H. Therefore, Home and foreign consumption are just the corresponding fraction of overall consumption:

$$P_t C_t = \frac{1}{\gamma} P_{H,t} C_{H,t} = \frac{1}{1-\gamma} P_{F,t} C_{F,t}$$

<u>Foreign country</u>

For F, the optimization problem is the same, except that

$$C_t^* = C_{H,t}^{*1-\gamma} C_{F,t}^{*\gamma}$$

This changes the first two first order condition with respect to Home and foreign good consumption:

$$\begin{aligned} L_{C_{H,t}}: \quad & \frac{1-\gamma}{C_{H,t}^*} && = \lambda_t^* P_{H,t}^* \\ L_{C_{F,t}}: \quad & \frac{\gamma}{C_{F,t}^*} && = \lambda_t^* P_{F,t}^* \end{aligned}$$

Those two first order conditions can be combined to:

$$\gamma^\gamma (1-\gamma)^{1-\gamma} = \lambda_t^* (P_{H,t}^* C_{H,t}^*)^{1-\gamma} (P_{F,t}^* C_{F,t}^*)^\gamma$$

which yields

$$\lambda_t^* = \frac{1}{P_t^* C_t^*}$$

where

$$P_t^* \equiv \frac{P_{H,t}^{*1-\gamma} P_{F,t}^{*\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$$

is defined as the utility-based price index in country F. Therefore, Home and foreign consumption are just the corresponding fraction of overall consumption:

$$P_t^* C_t^* = \frac{1}{1-\gamma} P_{H,t}^* C_{H,t}^* = \frac{1}{\gamma} P_{F,t}^* C_{F,t}^*$$

A.3.3 Intertemporal Allocation

Combining both consumption focs with the bond foc gives the Euler equation:

$$\frac{1}{C_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}}\right]$$

Now let's take a closer look at the financial market of the model. Let the variable $Q_{t,t+1}$ be the stochastic discount rate for j:

$$Q_{t,t+1} \equiv \beta \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

The expected stochastic discount factor is related to the inverse nominal interest rate (from the bond foc)

$$\mathbb{E}_t[Q_{t,t+1}] = \frac{1}{1+i_t} \qquad \mathbb{E}_t[Q_{t,t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}] = \frac{1}{1+i_t^*}\,.$$

In a symmetric model in which all agent can access the same domestic financial markets the individual discount factors are the same ($Q_{t,t+1} = Q_{t,t+1}$). Therefore, the nominal interest rates parity is given by:

$$(1+i_t) = \mathbb{E}_t\left[\frac{\mathcal{E}_{t+1}}{P_{t+1}C_{t+1}}\right]\mathbb{E}_t\left[\frac{\mathcal{E}_t}{P_{t+1}C_{t+1}}\right]^{-1}(1+i_t^*)$$

Finally, bonds are in zero net supply:

$$\int_0^1 B_{H,t-1}dj + \int_0^1 B^*_{H,t-1}dj^* = 0$$
$$\int_0^1 B_{F,t-1}dj + \int_0^1 B^*_{F,t-1}dj^* = 0$$

The first order condition for labor gives a condition that determines wages W_t for that period.
In addition, a transversality condition is imposed in order to ensure that consumers really exhaust their resources.

A.3.4 Prices

Firms selling brand h maximize profits:

$$\max\ \mathbb{E}_{t-1}[Q_{t-1,t}((1-\tau)p_t(h) - MC_t)\int_0^1 C_t(h,j)dj + (\frac{\mathcal{E}_t(1-\tau)\tilde{p}_t(h)}{\mathcal{E}_t} - (1+\varpi)MC_t))\int_0^1 C_t^*(h,j^*)dj^*)]$$

Accounting for consumer's demand (4) they choose prices such that they maximize their profits:

$$\max_{p_t(h),\tilde{p}_t(h)}\ \mathbb{E}_{t-1}[Q_{t-1,t}(((1-\tau)p_t(h) - MC_t)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} + ((1-\tau)\tilde{p}_t(h) - (1+\varpi)MC_t))\left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\right)^{-\theta} C^*_{H,t}]$$

For a firm the optimal domestic price is equal to marginal costs augmented by the equilibrium markup and an appropriate discount.

$$p_t(h) = \frac{1}{(1-\tau)}\frac{\theta}{\theta-1}\frac{\mathbb{E}_{t-1}[Q_{t-1,t}p_t(h)^{-\theta}P^{\theta}_{h,t}C_{H,t}MC_t]}{\mathbb{E}_{t-1}[Q_{t-1,t}p_t(h)^{-\theta}P^{\theta}_{h,t}C_{H,t}]}$$

Plugging in the stochastic discount rate and the relationship between expenditures for goods H and overall expenditures gives the price as in the main text:

$$p_t(h) = \frac{1}{(1-\tau)}\frac{\theta}{\theta-1}\mathbb{E}_{t-1}[MC_t]$$

The optimal price of Home goods in the foreign market can be obtained by differentiating the firm's objective function with respect to $\tilde{p}_t(h)$:

$$\mathbb{E}_{t-1}\left[Q_{t-1,t}\left((1-\tau)(1-\theta)\left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\right)^{-\theta}C^*_{H,t}+\theta(1+\varpi)MC_t\left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\right)^{-\theta}\tilde{p}_t(h)^{-1}C^*_{H,t}\right)\right] = 0$$

$$\mathbb{E}_{t-1}\left[Q_{t-1,t}\left((1-\tau)(\theta-1)\left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\right)^{-\theta}C^*_{H,t}\right)\right] = \mathbb{E}_{t-1}\left[Q_{t-1,t}\left(\theta(1+\varpi)MC_t\left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\right)^{-\theta}\tilde{p}_t(h)^{-1}C^*_{H,t}\right)\right]$$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[Q_{t-1,t}\tilde{p}_t(h)^{-\theta}\tilde{P}^{\theta}_{H,t}C^*_{H,t}MC_t]}{\mathbb{E}_{t-1}[Q_{t-1,t}\tilde{p}_t(h)^{-\theta}\tilde{P}^{\theta}_{H,t}C^*_{H,t}]}$$

Plug in the stochastic discount factor.

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{P_{t-1}C_{t-1}}{P_tC_t}\tilde{p}_t(h)^{-\theta}\tilde{P}^{\theta}_{H,t}C^*_{H,t}MC_t]}{\mathbb{E}_{t-1}[\frac{P_{t-1}C_{t-1}}{P_tC_t}\tilde{p}_t(h)^{-\theta}\tilde{P}^{\theta}_{H,t}C^*_{H,t}]}$$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{C^*_{H,t}}{P_tC_t}MC_t]}{\mathbb{E}_{t-1}[\frac{C^*_{H,t}}{P_tC_t}]}$$

Plug in demand for $C^*_{H,t}$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{(1-\gamma)P^*_tC^*_t/P^*_{H,t}}{P_tC_t}MC_t]}{\mathbb{E}_{t-1}[\frac{(1-\gamma)P^*_tC^*_t/P^*_{H,t}}{P_tC_t}]}$$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{C^*_t}{C_t}MC_t]}{\mathbb{E}_{t-1}[\frac{C^*_t}{C_t}]}$$

Consumption of both countries is always the same in a symmetric calibration, since terms of trade movements ensure perfect risk sharing. Therefore

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\mathbb{E}_{t-1}[MC_t]$$
$$p_t^*(h) = P_{H,t}^* = \frac{1}{(1-\tau)}(1+\varpi)\frac{\theta}{\theta-1}\frac{\mathbb{E}_{t-1}[MC_t]}{\mathcal{E}_t}$$

The firm then supplies for the given prices (wages and good prices) the amount of goods demanded by the consumers. This in the end determines the amount of work in the economy. With flexible prices, the expectations operator just drops and firms choose prices such that they match actual marginal costs, augmented with the equilibrium mark up.

A.3.5 Consumption

The first order condition of the consumer's problem yields, when optimizing w.r.t $C_{H,t}$ and $C_{F,t}$

$$1 = \lambda_t * P_{H,t}^{\gamma} P_{F,t}^{1-\gamma} \underbrace{(C_{H,t})^{\gamma}(C_{F,t})^{1-\gamma}}_{C_t} \underbrace{\frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}}_{\frac{1}{\gamma_w}}$$

This gives:

$$\lambda_t = \frac{1}{P_t C_t}$$

using the solution for the prices, consumption C_t is given by ($\lambda_t = 1/P_t C_t = 1/P_t C_t = 1/\mu_t$):

$$C_t = \frac{\gamma_w \mu_t}{P_{H,t}^{\gamma} P_{F,t}^{1-\gamma}}$$

or more explicit

$$C_t = (\frac{1}{1+\varpi})^{1-\gamma}\frac{\gamma_w \Phi^{-\gamma}\Phi^{*-(1-\gamma)}\mu_t \mathcal{E}_t^{-(1-\gamma)}}{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}$$
$$C_t^* = (\frac{1}{1+\varpi})^{1-\gamma}\frac{\gamma_w \Phi^{*-\gamma}\Phi^{*-(1-\gamma)}\mu_t^* \mathcal{E}_t^{1-\gamma}}{(\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{\gamma}}$$

A.3.6 Labor

The firm chooses labor such that it meets global demand for the brand:

$$L_t(h) = a_t^{-1}(p_t(h)^{-\theta} P_{H,t}^{\theta} C_{H,t} + p_t^*(h)^{-\theta}({P_{H,t}^*}^{\theta} C_{H,t}^*)$$

$MC_t = a_t^{-1} W_t$. Rearranging the labor foc, plugging in $\lambda = 1/\mu$ and you arrive at:

$$MC_t = a_t^{-1} \mu_t \kappa$$

In a symmetric equilibrium $p_t(h) = P_{H,t}$. Since households consume a constant fraction of foreign and Home goods ($P_t C_t \gamma = P_{H,t} C_{H,t}$), one can plug in $C_{H,t}$ and $C^*_{H,t}$ respectively to obtain:

$$L_t(h) = a_t^{-1} \left(\gamma \frac{\overbrace{P_t C_t}^{\mu_t}}{\underbrace{P_{H,t}}_{\Phi \mathbb{E}_{t-1}[MC_t]}} + (1-\gamma) \frac{P^*_t C^*_t}{P^*_{H,t}} \right)$$

Plugging in $P^*_{H,t}$ and the monetary stance and assuming that the degree of monopolistic distortion is the same in both countries

$$L_t(h) = \frac{1}{\Phi} a_t^{-1} \left(\gamma \frac{\mu_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi} \frac{\mu^*_t}{\frac{\mathbb{E}_{t-1}[MC_t]}{\mathcal{E}_t}} \right)$$

Augment the expression and use the relationship between both monetary stances and the exchange rate:

$$L_t(h) = \frac{1}{\Phi \kappa} \left(\gamma \frac{\overbrace{a_t^{-1} \kappa \mu_t}^{MC_t}}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi} \frac{a_t^{-1} \kappa \mu_t}{\mathbb{E}_{t-1}[MC_t]} \right)$$

Demand of for every good in F and H is a function of the marginal costs of the firm producing that good:

$$L_t(h) = \frac{1}{\Phi \kappa} \left(\gamma \frac{MC_t}{\mathbb{E}_{t-1}(MC_t)} + \frac{(1-\gamma)}{1+\varpi} \frac{MC_t}{\mathbb{E}_{t-1}(MC_t)} \right)$$
$$L^*_t(f) = \frac{1}{\Phi^* \kappa^*} \left(\frac{1-\gamma}{1+\varpi} \frac{MC^*_t}{\mathbb{E}_{t-1}(MC^*_t)} + \gamma \frac{MC^*_t}{\mathbb{E}_{t-1}(MC^*_t)} \right)$$

A.4 Solution Free Market and Flexible Prices

A.4.1 National Currency

The consumer solves the lifetime optimization problem. All variables can be expressed as a function of shocks a_t, a^*_t and economic parameter. The expectations operator drops when prices are flexible.

$$\mathcal{E}_t = \frac{\mu_t}{\mu_t^*} \tag{A.4}$$

$$MC_t = \kappa a_t^{-1} \mu_t \tag{A.5}$$

$$MC_t^* = \kappa^* a_t^{*-1} \mu_t^* \tag{A.6}$$

$$P_{H,t} = \Phi MC_t \tag{A.7}$$

$$P_{F,t} = \Phi^*(1+\varpi)\mathcal{E}_t MC_t^* \tag{A.8}$$

$$P_{F,t}^* = \Phi^* MC_t^* \tag{A.9}$$

$$P_{H,t}^* = \Phi(1+\varpi)\frac{1}{\mathcal{E}_t} MC_t \tag{A.10}$$

$$C_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi MC_t)^{\gamma} (\Phi^* MC_t^*)^{1-\gamma}} \tag{A.11}$$

$$C_t^* = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \mu_t^* \mathcal{E}_t^{1-\gamma}}{(\Phi MC_t)^{1-\gamma} (\Phi^* MC_t^*)^{\gamma}} \tag{A.12}$$

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma + \frac{(1-\gamma)}{1+\varpi}\right) \tag{A.13}$$

$$L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\left(\frac{1-\gamma}{1+\varpi} + \gamma\right) \tag{A.14}$$

A.4.2 Currency Union

$$\mathcal{E}_t = 1 \tag{A.15}$$

$$MC_t = \kappa a_t^{-1} \mu_t \tag{A.16}$$

$$MC_t^* = \kappa^* a_t^{*-1} \mu_t^* \tag{A.17}$$

$$P_{H,t} = \Phi MC_t \tag{A.18}$$

$$P_{F,t} = \Phi^* MC_t^* \tag{A.19}$$

$$P_{F,t}^* = \Phi^* MC_t^* \tag{A.20}$$

$$P_{H,t}^* = \Phi MC_t \tag{A.21}$$

$$C_t = \frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi MC_t)^{\gamma} (\Phi^* MC_t^*)^{1-\gamma}} \tag{A.22}$$

$$C_t^* = \frac{\gamma_w \mu_t^* \mathcal{E}_t^{1-\gamma}}{(\Phi MC_t)^{1-\gamma} (\Phi^* MC_t^*)^{\gamma}} \tag{A.23}$$

$$L_t(h) = \frac{1}{\Phi\kappa} \tag{A.24}$$

$$L_t^*(f) = \frac{1}{\Phi\kappa^*} \tag{A.25}$$

A.5 Solution Central Bank and Sticky Prices

A.5.1 National Currency

The consumer solves the lifetime optimization problem. All variables can be expressed as a function of shocks a_t, a_t^*, monetary stances μ_t, μ_t^* and economic parameter.

$$\mathcal{E}_t = \frac{\mu_t}{\mu_t^*} \tag{A.26}$$

$$MC_t = \kappa a_t^{-1} \mu_t \tag{A.27}$$

$$MC_t^* = \kappa^* a_t^{*-1} \mu_t^* \tag{A.28}$$

$$P_{H,t} = \Phi \mathbb{E}_{t-1}[MC_t] \tag{A.29}$$

$$P_{F,t} = \Phi^*(1+\varpi)\mathcal{E}_t \mathbb{E}_{t-1}[MC_t^*] \tag{A.30}$$

$$P_{F,t}^* = \Phi^* \mathbb{E}_{t-1}[MC_t^*] \tag{A.31}$$

$$P_{H,t}^* = \Phi(1+\varpi)\frac{1}{\mathcal{E}_t}\mathbb{E}_{t-1}[MC_t] \tag{A.32}$$

$$C_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi \mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^* \mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \tag{A.33}$$

$$C_t^* = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \mu_t^* \mathcal{E}_t^{1-\gamma}}{(\Phi \mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\Phi^* \mathbb{E}_{t-1}[MC_t^*])^{\gamma}} \tag{A.34}$$

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right) \tag{A.35}$$

$$L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\left(\frac{1-\gamma}{1+\varpi}\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} + \gamma \frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right) \tag{A.36}$$

A.5.2 Currency Union

$$\mathcal{E}_t = 1 \tag{A.37}$$

$$MC_t = \kappa a_t^{-1} \mu_t^U \tag{A.38}$$

$$MC_t^* = \kappa^* a_t^{*-1} \mu_t^U \tag{A.39}$$

$$P_{H,t} = \Phi \mathbb{E}_{t-1}[MC_t] \tag{A.40}$$

$$P_{F,t} = \Phi^* \mathcal{E}_t \mathbb{E}_{t-1}[MC_t^*] \tag{A.41}$$

$$P_{F,t}^* = \Phi^* \mathbb{E}_{t-1}[MC_t^*] \tag{A.42}$$

$$P_{H,t}^* = \Phi \frac{1}{\mathcal{E}_t} \mathbb{E}_{t-1}[MC_t] \tag{A.43}$$

$$C_t = \frac{\gamma_w \mu_t^U \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi \mathbb{E}_{t-1}[MC_t])^{\gamma} (\Phi^* \mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \tag{A.44}$$

$$C_t^* = \frac{\gamma_w \mu_t^U \mathcal{E}_t^{1-\gamma}}{(\Phi \mathbb{E}_{t-1}[MC_t])^{1-\gamma} (\Phi^* \mathbb{E}_{t-1}[MC_t^*])^{\gamma}} \tag{A.45}$$

$$L_t(h) = \frac{1}{\Phi \kappa} \left(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + (1-\gamma) \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} \right) \tag{A.46}$$

$$L_t^*(f) = \frac{1}{\Phi^* \kappa^*} \left(\gamma \frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} + (1-\gamma) \frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} \right) \tag{A.47}$$

A.6 Free Market and Flexible Prices

Now consider a decentralized economy, in which market forces determine the allocation. I show here that the flex price allocation is an important welfare benchmark. I consider two regimes, one with national currencies and one in a currency union.

A.6.1 National Currency

Households maximize their lifetime utility by choosing consumption and supplying labor:

$$\max_{\{C_t, L_t, B_t\}} \mathbb{E}_t \left[\sum_{j=t}^{\infty} \beta^j \left(\ln \left((C_{H,j})^{\gamma} (C_{F,}j)^{1-\gamma} \right) - \kappa L_j \right] \right.$$

$$\begin{aligned} \text{s.t.} \quad & B_{H,t} + \mathcal{E}_t B_{F,t} + P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = \\ & (1 + i_{t-1}) B_{H,t-1} - T_t + W_t L_t + (1 + i_{t-1}^*) \mathcal{E}_t B_{F,t-1} + \Pi_{H,t} \end{aligned}$$

Firms selling brand h maximize profits given the marginal costs, accounting for consumers' demand and the pricing strategy and trade costs with national currencies:

$$\max_{p_t(h), \tilde{p}_t(h)} \left((1-\tau) p_t(h) - MC_t \right) \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t} + \left(\frac{\mathcal{E}_t (1-\tau) \tilde{p}_t(h)}{\mathcal{E}_t} - (1+\varpi) MC_t \right) \left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}} \right)^{-\theta} C_{H,t}^*$$

The solution steps of that problem are in the appendix. Consumption and labor here have a superscript n:

$$\begin{aligned} C^n_{Ht} &= \frac{\gamma a_t}{\Phi\kappa} & C^{*n}_{Ht} &= \frac{(1-\gamma)\left(\frac{1}{1+\varpi}\right)a_t}{\Phi\kappa} \\ C^n_{Ft} &= \frac{(1-\gamma)\left(\frac{1}{1+\varpi}\right)a^*_t}{\Phi^*\kappa^*} & C^{*n}_{Ft} &= \frac{\gamma a^*_t}{\Phi^*\kappa^*} \\ L^n_t &= \frac{1}{\Phi\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) & L^{*n}_t &= \frac{1}{\Phi^*\kappa^*}\left(\frac{\gamma}{1+\varpi} + 1 - \gamma\right) \end{aligned} \tag{A.48}$$

The distribution of consumption in a decentralized allocation is the same, except that monopolistic markups lower consumption and employment, while trade costs lower consumption of non-domestic goods and overall employment.

A.6.2 Currency Union

Households face the same problem as before:

$$\begin{aligned} &\max_{\{C_t, L_t, B_t\}} \mathbb{E}_t\left[\sum_{j=t}^{\infty}\beta^j\left(\ln\left((C_{H,j})^\gamma (C_{F,j})^{1-\gamma}\right) - \kappa L_j\right]\right. \\ &\text{s.t. } B_{H,t} + \mathcal{E}_t B_{F,t} + P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = \\ &\qquad (1+i_t)B_{H,t-1} - T_t + W_t L_t + (1+i^*_t)\mathcal{E}_t B_{F,t-1} + \Pi_{H,t} \end{aligned}$$

In contrast to the case with national currencies, there are no trade costs and no exchange rate in a currency union:

$$\max_{p_t(h), p^*_t(h)} \left((1-\tau)p_t(h) - MC_t\right)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} + \left((1-\tau)p^*_t(h) - MC_t\right)\left(\frac{p^*_t(h)}{P^*_{H,t}}\right)^{-\theta} C^*_{H,t}$$

Consumption and labor are not a function of trade costs anymore and have superscript u:

$$\begin{aligned} C^u_{Ht} &= \frac{\gamma a_t}{\Phi\kappa} & C^{*u}_{Ht} &= \frac{(1-\gamma)a_t}{\Phi\kappa} \\ C^u_{Ft} &= \frac{(1-\gamma)a^*_t}{\Phi^*\kappa^*} & C^{*u}_{Ft} &= \frac{\gamma a^*_t}{\Phi^*\kappa^*} \\ L^u_t &= \frac{1}{\Phi\kappa} & L^{*u}_t &= \frac{1}{\kappa^*\Phi^*} \end{aligned} \tag{A.49}$$

Overall, employment and consumption in a currency union with flexible prices are the same as in the social planner's allocation, except for the monopolistic distortion.

A.7 Monetary Policy

For analytic convenience, let's introduce a monetary stance μ_t that controls nominal expenditures $P_t C_t$ in the economy. It links the nominal interest rate in the Euler equation such that

$$\frac{1}{\mu_t} = \beta(1+i_t)\mathbb{E}_t[\frac{1}{\mu_{t+1}}]$$

μ_{t+1}/μ_t determines the inflation target π, the steady state nominal interest rate is $1+i = \pi/\beta$. In equilibrium one obtains that $\mu_t = P_t C_t = W_t/\kappa$[20]. An expansionary monetary policy in H corresponds with interest rates cuts today or households' expectations about interest rate cuts in the future. In this case μ_t lies above the trend, it coincides with increased nominal spending $P_t C_t$ in the economy.

A.7.1 Optimal National Monetary Policy under Commitment

A national authority maximizes expected utility of the representative agent. I use a state-contingent notation:

$$\begin{aligned}
\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} & \Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta^{t-k}p(s^t \mid s^0)\Big(\ln(C_t)-\kappa L_t\Big)\Big] \\
\text{s.t.} C_t &= (\frac{1}{1+\varpi})^{1-\gamma}\frac{\gamma_w\big(\frac{\theta-1}{\theta}\big)^{\gamma}\big(\frac{\theta^*-1}{\theta^*}\big)^{1-\gamma}\mu_t(s^t)\mathcal{E}_t^{-1(1-\gamma)}}{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \\
L_t(h) &= \frac{\theta-1}{\theta\kappa}\Big(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}+\frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\Big) \\
MC_t &= \kappa a_t^{-1}\mu_t(s^t) \\
MC_t^* &= \kappa^* a_t^{*-1}\mu_t^*(s^t) \\
\mathcal{E}_t &= \frac{1-\gamma}{\gamma}\frac{\mu_t(s^t)}{\mu_t^*(s^t)} \\
\mathbb{E}_{t-1}[MC_t] &= \sum_{s^t\in A}p(s^t \mid s^0)MC_t \\
\mathbb{E}_{t-1}[MC_t^*] &= \sum_{s^t\in A}p(s^t \mid s^0)MC_t^*
\end{aligned}$$

Plugging in:

$$\begin{aligned}
\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} & \Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta^{t-k}p(s^t \mid s^0) \\
&\Big(\ln((\frac{1}{1+\varpi})^{1-\gamma}\frac{\gamma_w\big(\frac{\theta-1}{\theta}\big)^{\gamma}\big(\frac{\theta^*-1}{\theta^*}\big)^{1-\gamma}\mu_t(s^t)(\frac{1-\gamma}{\gamma}\frac{\mu_t(s^t)}{\mu_t^*(s^t)})^{-1(1-\gamma)}}{(\sum_{s^t\in A}p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t))^{\gamma}(\sum_{s^t\in A}p(s^t \mid s^0)\kappa^* a_t^{*-1}\mu_t^*(s^t))^{1-\gamma}}) \\
&-\kappa\frac{\theta-1}{\theta\kappa}\Big(\gamma\frac{\kappa a_t^{-1}\mu_t(s^t)}{\sum_{s^t\in A}p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)}+\frac{(1-\gamma)}{1+\varpi}\frac{\kappa a_t^{-1}\mu_t(s^t)}{\sum_{s^t\in A}p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)}\Big)\Big)\Big]
\end{aligned}$$

[20] Inspect the Euler equation with logarithmic utility for that

Dissolve the ln expression

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \sum_{s^t\in A} p(s^t \mid s^0)\Bigg[\ln\left((\frac{1}{1+\varpi})^{1-\gamma}\gamma_w(\frac{\theta-1}{\theta})^{\gamma}(\frac{\theta^*-1}{\theta^*})^{1-\gamma}(\frac{1-\gamma}{\gamma})^{-(1-\gamma)}\right)+\ln(\mu_t(s^t))$$

$$-(1-\gamma)(\ln(\mu_t(s^t))-\ln(\mu_t^*(s^t)))-\gamma\ln(\sum_{s^t\in A} p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t))$$

$$-(1-\gamma)\ln(\sum_{s^t\in A} p(s^t \mid s^0)\kappa^* a_t^{*-1}\mu_t^*(s^t))\Bigg]$$

$$-\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\left[\kappa\frac{\theta-1}{\theta\kappa}\left(\gamma\frac{\kappa a_t^{-1}\mu_t(s^t)}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa a_t^{-1}\mu_t(s^t)}+\frac{1-\gamma}{1+\varpi}\frac{\kappa a_t^{-1}\mu_t(s^t)}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa a_t^{-1}\mu_t(s^t)}\right)\right].$$

Note, that the last term representing labor is just a constant under monetary policy under commitment, the first order condition is

$$\frac{1}{\mu_t(s^t)}-\frac{(1-\gamma)}{\mu_t(s^t)}-\gamma\frac{\kappa a_t^{-1}}{\sum_{s^t\in A} p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)}=0$$

This can be rewritten to get the optimal monetary policy as in the main text:

$$\frac{1}{\mu_t(s^t)}=\frac{\kappa a_t^{-1}}{\sum_{s^t\in A} p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)}$$
$$\mu_t(s^t)a_t^{-1}(s^t)=\mathbb{E}_{t-1}\left[\mu_t(s^t)a_t^{-1}(s^t)\right]$$

Alternatively, we can also use the utility gap approach as in Corsetti and Pesenti (2005)

$$\min \mathbb{E}_{t-1}[W_t^{flex}-W_t]=\min \mathbb{E}_{t-1}\left[\ln\left(C_t^{flex}/C_t\right)-\kappa L_t^{flex}+\kappa L_t\right]$$

$$\min \mathbb{E}_{t-1}\left[\ln\frac{(\left(\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w\mu_t\mathcal{E}_t^{-1(1-\gamma)}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)}{\left(\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w\mu_t\mathcal{E}_t^{-1(1-\gamma)}}{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}\right))}-\kappa L_t^{flex}+\kappa L_t\right]$$

$$\min \mathbb{E}_{t-1}\left[\ln\left(\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)\right)-\kappa L_t^{flex}+\kappa L_t\right]$$

Now plug in labor

$$\min \mathbb{E}_{t-1}\left[\ln\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)-\frac{1}{\kappa}\left(\gamma+\frac{1-\gamma}{1+\varpi}\right)+\kappa\frac{1}{\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}+\frac{(1-\gamma)}{1+\varpi}\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)\right]$$
$$\min \mathbb{E}_{t-1}\left[\ln\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)\right]-\frac{1}{\kappa}\left(\gamma+\frac{1-\gamma}{1+\varpi}\right)+\kappa\frac{1}{\kappa}\left(\gamma\frac{\mathbb{E}_{t-1}MC_t}{\mathbb{E}_{t-1}[MC_t]}+\frac{(1-\gamma)}{1+\varpi}\frac{\mathbb{E}_{t-1}MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)$$

Under Monetary Policy under commitment, labor is not actively targeted for by monetary policy and trade costs do not play a role. Therefore, monetary policy optimally minimizes:

$$\min \mathbb{E}_{t-1}\Bigg[\ln\Big(\Big(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\Big)\Big)\Bigg]$$

Note that, according to Jensen's Inequality

$$\begin{aligned}
&\ln\Big((\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}\Big) - \mathbb{E}_{t-1}\Big[\ln(MC_t^{\gamma}(MC_t^*)^{1-\gamma})\Big] \\
\geq\ &\mathbb{E}_{t-1}\Big[\ln((MC_t])^{\gamma}([MC_t^*])^{1-\gamma}\Big] - \mathbb{E}_{t-1}\Big[\ln(MC_t^{\gamma}(MC_t^*)^{1-\gamma})\Big] = 0
\end{aligned}$$

The best monetary policy could do is to set the gap to 0. Rewrite the objective function to:

$$\begin{aligned}
&\min \mathbb{E}_{t-1}\Bigg[\gamma\ln\Big(\frac{\mathbb{E}_{t-1}[MC_t]}{MC_t}\Big) + (1-\gamma)\ln\Big(\frac{(\mathbb{E}_{t-1}[MC_t^*]}{MC_t^*}\Big)\Bigg] \\
=&\min_{\mu_t} \mathbb{E}_{t-1}\Bigg[\gamma\ln\Big(\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}\mu_t}\Big) \\
=&\min_{\mu_t} \sum_{s^t\in A} p(s^t \mid s^0)\Bigg[\gamma\ln\Big(\frac{\sum_{s^t\in A} p(s^t \mid s^0)[a_t^{-1}\mu_t]}{a_t^{-1}\mu_t}\Big)\Bigg] \\
=&\min_{\mu_t} \sum_{s^t\in A} p(s^t \mid s^0)\Bigg[\gamma(\ln(\sum_{s^t\in A} p(s^t \mid s^0)[a_t^{-1}\mu_t]) - \ln(a_t^{-1}\mu_t))\Bigg]
\end{aligned}$$

Differentiate for specific state $\bar{A}$, then the first order condition is:

$$p(\bar{A})\Bigg[\frac{a_t^{-1}(\bar{A})}{\sum_{s^t\in A} p(s^t \mid s^0)a_t^{-1}(s_t)\mu_t(s_t)}(\sum_{s^t\in A} p(s^t \mid s^0))\Bigg] - p(\bar{A})\Bigg[\frac{a_t^{-1}(\bar{A})}{a_t^{-1}(\bar{A})\mu_t(\bar{A})}\Bigg] = 0$$

The policy rule for state $\bar{A}$ is:

$$a_t^{-1}(\bar{A})\mu_t(\bar{A}) = \mathbb{E}_{t-1}[a_t^{-1}\mu_t]$$

The same can be done for the foreign country. Note that under commitment, the central bank can not resort to negative monetary surprises to push the gap below zero.

We can also differentiate with respect to μ_t making use of the result: $\frac{\partial f(\mathbb{E}_{t-1}[x_t\mu_t^{\pi}])}{\partial \mu_t} = f'(\mathbb{E}_{t-1}[x_t\mu_t^{\pi}]) \cdot x_t\pi\mu_t^{\pi-1}$

$$\begin{aligned}
0 &= \frac{1}{\mu_t} - \frac{(1-\gamma)}{\mu_t} - \gamma\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} \\
\Rightarrow \mu_t &= \frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}
\end{aligned}$$

This way we can avoid the state contingent notation.

Alternative version: Try to avoid using μ_t as a policy instrument and add time discount shock:

$$
\begin{aligned}
\max_{\{i_t(s^t)\}_{t=k}^{\infty}} & \left[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta_t^{t-k} p(s^t \mid s^0) \Big(\ln(C_t) - \kappa L_t \Big) \right] \\
\text{s.t.} C_t &= \mu_t^{-1}/P_t \\
\mu_t &= (\beta_t(1+i_t)) \left(\mathbb{E}_t \left[\frac{1}{P_{t+1}C_{t+1}} \right] \right) \\
P_t &= \frac{P_{H,t}^{\gamma} P_{F,t}^{1-\gamma}}{\gamma_w} \\
L_t(h) &= \frac{\theta-1}{\theta\kappa} \left(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi} \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} \right) \\
P_{H,t} &= \Phi \mathbb{E}_{t-1}[MC_t] \\
P_{F,t} &= \Phi^* \mathcal{E}_t (1+\varpi) \mathbb{E}_{t-1}[MC_t^*] \\
MC_t &= \kappa a_t^{-1} \mu_t(s^t)^{-1} \\
MC_t^* &= \kappa^* a_t^{*-1} \mu_t^*(s^t)^{-1} \\
\mathcal{E}_t &= \left(\frac{\mu_t(s^t)}{\mu_t^*(s^t)} \right)^{-1} \\
\mathbb{E}_{t-1}[MC_t] &= \sum_{s^t \in A} p(s^t \mid s^0) MC_t \\
\mathbb{E}_{t-1}[MC_t^*] &= \sum_{s^t \in A} p(s^t \mid s^0) MC_t^*
\end{aligned}
$$

Plugging in everything except the Euler equation and considering the expectations operator in front:

$$
\begin{aligned}
&\max_{\{i_t(s^t)\}_{t=k}^{\infty}} \left[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta_t^{t-k} p(s^t \,|\, s^{t-1}) \left(\ln \left(\frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma} \gamma_w \left(\frac{\theta-1}{\theta}\right)^{\gamma} \left(\frac{\theta^*-1}{\theta^*}\right)^{1-\gamma} \mu_t(s^t)^{-\gamma} \mu_t^*(s^t)^{-1+\gamma}}{(\mathbb{E}_{t-1}\left[\kappa a_t^{-1} \mu_t(s^t)^{-1}\right])^{\gamma} (\mathbb{E}_{t-1}\left[\kappa a_t^{*-1} \mu_t^*(s^t)^{-1}\right])^{1-\gamma}} \right) \right) \right] \\
&\text{s.t.} \quad \mu_t = (\beta_t(1+i_t)) \left(\mathbb{E}_t \left[\frac{1}{P_{t+1}C_{t+1}} \right] \right) \\
&\qquad\;\; \mu_t^* = (\beta_t^*(1+i_t^*)) \left(\mathbb{E}_t \left[\frac{1}{P_{t+1}^* C_{t+1}^*} \right] \right)
\end{aligned}
$$

Plugging in both Euler equation, I obtain the following maximization problem

$$
\begin{aligned}
&\max_{\{i_t(s^t)\}_{t=k}^{\infty}} \left[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta_t^{t-k} p(s^t \,|\, s^{t-1}) \right. \\
&\left. \ln \left(\frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma} \gamma_w (\Phi)^{\gamma} (\Phi^*)^{1-\gamma} \left((\beta_t(1+i_t)) \left(\mathbb{E}_t \left[\frac{1}{P_{t+1}C_{t+1}} \right] \right) \right)^{-\gamma} \left((\beta_t^*(1+i_t^*)) \left(\mathbb{E}_t \left[\frac{1}{P_{t+1}^* C_{t+1}^*} \right] \right) \right)^{\gamma-1}}{\left(\mathbb{E}_{t-1} \left[\kappa a_t^{-1} \left(\beta_t(1+i_t) \mathbb{E}_t \left[\frac{1}{P_{t+1}C_{t+1}} \right] \right)^{-1} \right] \right)^{\gamma} \left(\mathbb{E}_{t-1} \left[\kappa a_t^{*-1} \left(\beta_t^*(1+i_t^*) \mathbb{E}_t \left[\frac{1}{P_{t+1}^* C_{t+1}^*} \right] \right)^{-1} \right] \right)^{1-\gamma}} \right) \right]
\end{aligned}
$$

In an iid case $\left(\mathbb{E}_t\left[\frac{1}{P^*_{t+1}C^*_{t+1}}\right]\right)$ cancels out. We are left with:

$$\max_{\{i_t(s^t)\}_{t=k}^{\infty}} \left[\sum_{t=k}^{\infty}\sum_{s^t\in A} \beta_t^{t-k} p(s^t\,|\,s^{t-1}) \ln\left(\frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w(\Phi)^{\gamma}(\Phi^*)^{1-\gamma}\left((\beta_t(1+i_t))\right)^{-\gamma}\left((\beta^*_t(1+i^*_t))\right)^{\gamma-1}}{\left(\mathbb{E}_{t-1}\left[\kappa a_t^{-1}(\beta_t(1+i_t))^{-1}\right]\right)^{\gamma}\left(\mathbb{E}_{t-1}\left[\kappa a_t^{*-1}(\beta^*_t(1+i^*_t))^{-1}\right]\right)^{1-\gamma}}\right)\right]$$

Derivative with respect to i_t:

$$-\gamma\frac{1}{(1+i_t)} + \gamma\frac{\kappa a_t^{-1}\beta_t^{-1}\frac{1}{1+i_t}^{2}}{\mathbb{E}_{t-1}\left[\kappa a_t^{-1}(\beta_t(1+i_t))\right]} = 0$$

The monetary interest rate rule is described by:

$$a_t^{-1}\beta_t^{-1}(1+i_t)^{-1} = \mathbb{E}_{t-1}\left[a_t^{-1}(\beta_t(1+i_t))^{-1}\right]$$

Supply shock. a_t goes up (expansionary). implies that country is more productive. Central bank optimally lowers interest rates.

Demand shock. β_t goes up (contractionary). implies that households want to save more. Central bank optimally lowers interest rates.

A.7.2 Optimal Monetary Policy in a Currency Union under Commitment

Now take a look at the monetary optimization problem:

$$\min \xi\left(\mathbb{E}_{t-1}\left[\ln\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC^*_t])^{1-\gamma}}{MC_t^{\gamma}(MC^*_t)^{1-\gamma}}\right)\right]\right)+(1-\xi)\left(\mathbb{E}_{t-1}\left[\ln\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\mathbb{E}_{t-1}[MC^*_t])^{\gamma}}{MC_t^{1-\gamma}(MC^*_t)^{\gamma}}\right)\right]\right)$$
$$\min \xi\left(\mathbb{E}_{t-1}\left[\gamma\ln\left(\frac{\mathbb{E}[MC_t]}{MC_t}\right)+(1-\gamma)\ln\left(\frac{\mathbb{E}[MC^*_t]}{MC^*_t}\right)\right]+(1-\xi)\left(\mathbb{E}_{t-1}\left[(1-\gamma)\ln\left(\frac{\mathbb{E}[MC_t]}{MC_t}\right)+\gamma\ln\left(\frac{\mathbb{E}[MC^*_t]}{MC^*_t}\right)\right]\right)\right)$$
$$\min \mathbb{E}\left[\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\ln\left(\frac{\mathbb{E}[MC_t]}{MC_t}\right)+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\ln\left(\frac{\mathbb{E}[MC^*_t]}{MC^*_t}\right)\right]$$

Weights do not matter if $\gamma = 1/2$, every country values Home and foreign goods equally. The state contingent objective function is

$$\min \sum_{s^t\in A} p(s^t \mid s^0)\Bigg[\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\ln\left(\frac{\sum_{s^t\in A}p(s^t\mid s^0)[a_t^{-1}(s^t)\mu_t(s^t)]}{a_t^{-1}(s^t)\mu_t(s^t)}\right)$$
$$+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\ln\left(\frac{\sum_{s^t\in A}p(s^t\mid s^0)[a_t^{*-1}(s^t)\mu_t(s^t)]}{a_t^{*-1}(s^t)\mu_t(s^t)}\right)\Bigg]$$

The first order condition with respect to $\mu_t(\bar{A})$ is

$$\big(\xi\gamma+(1-\xi)(1-\gamma)\big)p(\bar{A})\Big[\frac{a_t^{-1}(\bar{A})}{\sum_{s^t\in A}p(s^t\,|\,s^{t-1})a_t^{-1}(s_t)\mu_t(s_t)}(\sum_{s^t\in A}p(s^t\mid s^0))-\frac{a_t^{-1}(\bar{A})}{a_t^{-1}(\bar{A})\mu_t(\bar{A})}\Big]$$
$$+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)p(\bar{A})\Big[\frac{a_t^{*-1}(\bar{A})}{\sum_{s^t\in A}p(s^t\,|\,s^{t-1})a_t^{*-1}(s_t)\mu_t(s_t)}(\sum_{s^t\in A}p(s^t\,|\,s^{t-1}))-\frac{a_t^{*-1}(\bar{A})}{a_t^{*-1}(\bar{A})\mu_t(\bar{A})}\Big]=0$$

Solving for $\mu_t(\bar{A})$:

$$\mu_t(\bar{A})=\Bigg(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\Bigg)^{-1}$$

For symmetric consumption baskets without Home bias as in Corsetti and Pesenti (2002), the objective functions boils down to:

$$\min_{\mu_t}\sum_{s^t\in A}p(s^t\mid s^0)\Big[\gamma(\ln(\sum_{s^t\in A}p(s^t\mid s^0)[a_t^{-1}\mu_t])-\ln(a_t^{-1}\mu_t))\Big]$$
$$+\sum_{s^t\in A}p(s^t\mid s^0)\Big[(1-\gamma)(\ln(\sum_{s^t\in A}p(s^t\mid s^0)[a_t^{*-1}\mu_t])-\ln(a_t^{*-1}\mu_t))\Big]$$

giving the same optimal monetary stance as in Corsetti and Pesenti (2002).

Maximizing expected lifetime utility ex ante leads to the same monetary rule:

$$\max_{\mu_s}\xi\sum\beta^t\Big(\sum p_s\Big[\ln(C_s)-\kappa L_s\Big]\Big)+(1-\xi)\sum\beta^t\Big(\sum p_s\Big[\ln(C_s^*)-\kappa l_s^*\Big]\Big)$$

Plugging in consumption and labor, the foc for the monetary stance is:

$$\xi p_s\Big[\frac{1}{\mu_s}-\frac{\gamma\kappa a^{-1}}{\mathbb{E}[MC_t]}-\frac{(1-\gamma)\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}\Big]+(1-\xi)p_s\Big[\frac{1}{\mu_s}-\frac{(1-\gamma)\kappa a^{-1}}{\mathbb{E}[MC_t]}-\frac{(\gamma\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}\Big]$$
$$+\xi\sum_{p_A^-}(\frac{-\gamma p_s\kappa a^{-1}}{\mathbb{E}[MC_t]}-\frac{(1-\gamma)p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]})+(1-\xi)\sum_{p_A^-}(\frac{-(1-\gamma)p_s\kappa a^{-1}}{\mathbb{E}[MC_t]}-\frac{\gamma p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]})=0$$

Rearranging a bit gives

$$\frac{\xi p_s}{\mu_s}+\frac{(1-\xi)p_s}{\mu_s}-\frac{\xi\gamma p_s\kappa a^{-1}}{\mathbb{E}[MC_t]}-\frac{(1-\xi)(1-\gamma)p_s\kappa a^{-1}}{\mathbb{E}[MC_t]}-\frac{\xi(1-\gamma)p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}-\frac{(1-\xi)\gamma p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}=0$$
$$\mu_t(\bar{A})=\Bigg(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\Bigg)^{-1}$$

Avoid using μ_t and introduce demand shocks:

$$
\begin{aligned}
&\max_{\{i_t(s^t)\}_{t=k}^{\infty}} \xi\Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s_t\,|\,s^{t-1})\Big(\ln(C_t)-\kappa L_t\Big)\Big] + (1-\xi)\Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s_t\,|\,s^{t-1})\Big(\ln(C_t)-\kappa L_t\Big)\Big]\\
&\text{s.t.}C_t = \lambda_{1,t}^{-1}/P_t \qquad C_t^* = \lambda_{1,t}^{*-1}/P_t^*\\
&\lambda_{1t} = (\beta_t(1+i_t))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}C_{t+1}}\right]\right) \qquad \lambda_{1t}^* = (\beta_t^*(1+i_t))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}^*C_{t+1}^*}\right]\right)\\
&P_t = \frac{P_{H,t}^{\gamma}P_{F,t}^{1-\gamma}}{\gamma_w} \qquad P_t^* = \frac{P_{H,t}^{*1-\gamma}P_{F,t}^{*\gamma}}{\gamma_w}\\
&L_t(h) = \frac{\theta-1}{\theta\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + (1-\gamma)\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right)\\
&P_{H,t} = \Phi\mathbb{E}_{t-1}[MC_t]\\
&P_{F,t} = \Phi^*\mathbb{E}_{t-1}[MC_t^*]\\
&MC_t = \kappa a_t^{-1}\lambda_{1,t}^{-1}\\
&MC_t^* = \kappa^* a_t^{*-1}\lambda_{1,t}^{*-1}\\
&\mathbb{E}_{t-1}[MC_t] = \sum_{s^t\in A} p(s^t \mid s^0)MC_t\\
&\mathbb{E}_{t-1}[MC_t^*] = \sum_{s^t\in A} p(s^t \mid s^0)MC_t^*
\end{aligned}
$$

A.7.3 Optimal Discretion with National Currencies

Now consider optimal monetary policy under discretion, the monetary authority maximizes

$$
\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}}\left[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta^{t-k}p(s^t \mid s^t)\Big(\ln(C_t)-\kappa L_t\Big)\right]
$$

The decisive difference to the optimization problem before is that the information set (inside the probability function) is for period t not $t-1$. The problem is subject to all equilibrium conditions. Plugging these in as before, the central bank has to maximize

$$
\begin{aligned}
\max_{\mu_t(s^t)} \quad & \ln\left(\Big(\frac{1}{1+\varpi}\Big)^{1-\gamma}\gamma_w\big(\Phi\big)^{-\gamma}\big(\Phi\big)^{-(1-\gamma)}\Big(\frac{1-\gamma}{\gamma}\Big)^{-(1-\gamma)}\right) + \ln(\mu_t(s^t))\\
& -(1-\gamma)(\ln(\mu_t(s^t)) - \ln(\mu_t^*(s^t))) - \gamma\ln\Big(\sum_{s^t\in A} p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)\Big)\\
& -(1-\gamma)\ln\Big(\sum_{s^t\in A} p(s^t \mid s^0)\kappa^* a_t^{*-1}\mu_t^*(s^t)\Big)\\
& -\kappa\frac{1}{\Phi\kappa}\frac{1+\gamma\varpi}{1+\varpi}\left(\frac{\kappa a_t^{-1}\mu_t(s^t)}{\sum_{s^t\in A}p(s^t\,|\,s^{t-1})\kappa a_t^{-1}\mu_t(s^t)}\right).
\end{aligned}
$$

In this case labor is not just a constant and the focs with respect to monetary policy in state $\bar{A}$ are

$$\frac{1}{\mu_t(\bar{A})} - \frac{(1-\gamma)}{\mu_t(\bar{A})} - \gamma \frac{\kappa a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)}$$
$$-\kappa \frac{1}{\Phi \kappa} \frac{1+\gamma\varpi}{1+\varpi} \left[\frac{\kappa a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)} - \frac{\kappa a_t(\bar{A})^{-1} \mu_t(\bar{A}) \kappa a_t(\bar{A})^{-1}}{\left(\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t) \right)^2} \right] = 0$$

Rearrange to get

$$\frac{\gamma}{\mu_t(\bar{A})} - \gamma \frac{a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)}$$
$$= \underbrace{\frac{1+\gamma\varpi}{\Phi(1+\varpi)}}_{\Theta^N} \frac{a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)} \left[1 - \frac{a_t(\bar{A})^{-1} \mu_t(\bar{A})}{\sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)} \right]$$

The solution of this problem in general differs from $a_t^{-1}\mu_t(s^t) = \sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)$. Rearrange a bit and use the notation with the expectation operator again:

$$\frac{\gamma}{\mu_t(\bar{A})} = \gamma \frac{a_t(\bar{A})^{-1}}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} + \Theta^N \frac{a_t(\bar{A})^{-1}}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} \left[1 - \frac{a_t(\bar{A})^{-1} \mu_t(\bar{A}))}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} \right]$$
$$\Rightarrow \frac{\gamma}{\Theta^N} = \left(1 - \frac{a_t(\bar{A})^{-1} \mu_t(\bar{A})}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} + \frac{\gamma}{\Theta^N} \right) \frac{a_t(\bar{A})^{-1}}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)}$$

Optimal monetary policy in state $\bar{A}$ is hence characterized by

$$\frac{a_t(\bar{A})^{-1} \mu_t(\bar{A})}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} = \frac{\gamma}{\Theta^N}$$

If

$$\frac{\gamma}{\Theta^N} = 1 \qquad \text{then} \qquad a_t^{-1} \mu_t(s^t) = \mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)$$

There is no bias in the monetary policy decision rule and output and employment gaps are closed. Even under discretion monetary policy puts the economy to its first best. If

$$\frac{\gamma}{\Theta^N} > 1, \qquad \text{then} \qquad a_t^{-1} \mu_t(s^t) > \mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)$$

Monetary policy has an inflationary bias, as the size of the domestic economy (or the preference for domestic goods consumption, depending on your interpretation) γ is so great, that the central bank cares more about domestic markups. If

$$\frac{\gamma}{\Theta^N} < 1 \qquad \text{then} \qquad a_t^{-1} \mu_t(s^t) < \mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)$$

Monetary policy has a deflationary bias. The domestic economy is relatively unimportant for consumers welfare and the central bank tries to make foreign goods cheaper via terms of trade movements. (explanation via markups is analogous, noting that smaller values for $\Theta^N < 1$ imply higher markups.

The reason for the bias under monetary policy under discretion is that firms anticipate that monetary policy wants to use surprise policies. In the case of an inflationary bias monetary policy tries to inflate away the domestic markup when firms cannot react anymore. Anticipating that, domestic firms already increase the price before. The deflationary bias stems from the desire of the central bank to use surprise terms of trade movements to make non-domestic goods cheaper. Under PCP foreign firms still receive the same price, but domestic consumers have to pay less.

A.7.4 Optimal Discretion in a Union

Now consider the central bank in F, that acts under discretion. This means that the information set of the expectation operator in the maximization problem is for period t and not for period $t-1$. The objective function is therefore:

$$\min \mathbb{E}_t[W_t^{flex} - W_t] = \min \mathbb{E}_t \left[\ln\left(C_t^{flex}/C_t\right) - \kappa L_t^{flex} + \kappa L_t\right]$$

Under discretion monetary policy is characterized by the following rule:

$$\mu_t^* = \frac{\gamma}{\Theta^{*N}} \frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}$$

where $\frac{\gamma}{\Theta^{*N}}$ is a bias[21] stemming from discretionary policy. As discussed by Corsetti and Pesenti (2001), this bias can either be inflationary or deflationary. If $\frac{\gamma}{\Theta^{*N}} = 1$, then there is no bias, if $\frac{\gamma}{\Theta^{*N}} > 1$ there is an inflationary bias. As domestic markups are very important for the welfare of the agents in the economy, the central bank tries to inflate away the monopolistic markups. That is, when Θ^{*N} is small and/or when γ is very large. In contrast a deflationary bias arises, if $\frac{\gamma}{\Theta^{*N}} < 1$. In that case domestic markups and domestic goods in general are less important and the central bank tries to deflate the value of the currency such that domestic consumers can buy more non-domestic goods. This case is in particular relevant, if γ is low. That is if consumers have a strong preference for non-domestic goods.

A common central bank maximizes a weighted sum of both countries' lifetime utility

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \left[\xi \sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^t) \Big(\ln(C_t) - \kappa L_t \Big) + (1-\xi) \sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^t) \Big(\ln(C_t^*) - \kappa^* l_t^* \Big) \right]$$

[21] $\Theta^{*N} = \frac{1+\gamma\varpi}{\Phi^*(1+\varpi)}$, $\Phi^* = \frac{\theta^*}{(\theta^*-1)(1-\tau)}$

subject to the equilibrium conditions in a currency union:

$$\begin{aligned}
MC_t &= \kappa a_t^{-1}\mu_t^U \\
MC_t^* &= \kappa^* a_t^{*-1}\mu_t^U \\
C_t &= \frac{\gamma_w \mu_t^U \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \\
C_t^* &= \frac{\gamma_w \mu_t^U \mathcal{E}_t^{1-\gamma}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{\gamma}} \\
L_t(h) &= \frac{1}{\kappa\Phi}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + (1-\gamma)\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right) \\
L_t^*(f) &= \frac{1}{\kappa^*\Phi^*}\left(\gamma\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} + (1-\gamma)\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)
\end{aligned}$$

Recall that $\Phi\frac{1+\gamma\varpi}{1+\varpi} = \Theta^N$ and let $\Phi = \Theta^U$. As the markups in the union do not contain any trade costs $\Theta^U < \Theta^N$. The central bank maximizes

$$\begin{aligned}
\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \Bigg[\xi\Bigg(&\ln\Big(\frac{\gamma_w\mu_t^U}{(\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa a_t^{-1}\mu_t^U)^{\gamma}(\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa^* a_t^{*-1}\mu_t^U)^{1-\gamma}}\Big) \\
&- \Theta^U\Big(\frac{\kappa a_t^{-1}\mu_t^U}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa a_t^{-1}\mu_t^U}\Big)\Bigg) \\
&+(1-\xi)\Bigg(\ln\Big(\frac{\gamma_w\mu_t^U}{(\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa a_t^{-1}\mu_t^U)^{1-\gamma}(\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa^* a_t^{*-1}\mu_t^U)^{\gamma}}\Big) \\
&-\Theta^{*U}\Big(\frac{\kappa^* a_t^{*-1}\mu_t^U}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})\kappa^* a_t^{*-1}\mu_t^U}\Big)\Bigg)\Bigg]
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
&\xi\Bigg[\frac{1}{\mu(\bar{A})} - \frac{\gamma a_t^{-1}(\bar{A})}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{-1}\mu_t^U} - \frac{(1-\gamma)a_t^{*-1}(\bar{A})}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{*-1}\mu_t^U} \\
&- \Theta^U\left(\frac{a_t^{-1}(\bar{A})}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{-1}\mu_t^U} - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})a_t^{-1}(\bar{A})}{(\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{-1}\mu_t^U)^2}\right) \\
&+ (1-\xi)\Bigg[\frac{1}{\mu(\bar{A})} - \frac{(1-\gamma)a_t^{-1}(\bar{A})}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{-1}\mu_t^U} - \frac{\gamma a_t^{*-1}(\bar{A})}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{*-1}\mu_t^U} \\
&- \Theta^{*U}\left(\frac{a_t^{*-1}(\bar{A})}{\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{*-1}\mu_t^U} - \frac{a_t^{*-1}(\bar{A})\mu_t(\bar{A})a_t^{*-1}(\bar{A})}{(\sum_{s^t\in A} p(s^t\,|\,s^{t-1})a_t^{*-1}\mu_t^U)^2}\right) = 0
\end{aligned}$$

Rearrange and compare to the solution before

$$\begin{aligned}
\frac{1}{\mu_t(\bar{A})} &= \big(\xi\gamma + (1-\xi)(1-\gamma)\big)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma) + (1-\xi)\gamma\big)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]} \\
&+ \Theta^U\xi\left(1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \Theta^{*U}(1-\xi)\left(1 - \frac{a_t^{*-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\right)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}
\end{aligned}$$

The first row is the same as under commitment in a monetary union, while the second one represents the inflationary or deflationary bias .

Consider a state where both countries have the same productivity: $a_t(\bar{A}) = a_t^*(\bar{A})$,

$$\frac{1}{\mu_t(\bar{A})} = \frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \left(\xi\Theta^U + (1-\xi)\Theta^{*U}\right)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\left[1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right]$$

This can be rearranged in the same way as before for $\Theta^U = \Theta^{*U}$

$$\frac{1}{\Theta^U} = \frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\left[1 - \frac{a_t^{-1}\mu_t}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \frac{1}{\Theta^U}\right]$$

The solution is

$$\frac{1}{\Theta^U} = \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}a_t^{-1}\mu_t}$$

Compare this to the discretionary monetary policy in H outside the union:

$$\frac{\gamma}{\Theta^N} = \frac{a_t(\bar{A})^{-1}\mu_t(\bar{A})}{\mathbb{E}_{t-1}a_t^{-1}\mu_t}$$

For the first best allocation we know that the LHS must be one. We know that $\Theta^N < \Theta^U$, but $\gamma < 1$. This means that there are only gains of a union, if the drop in markups is sufficiently large. As the deflationary bias stemming from incentives to manipulate the exchange rate is removed, the mitigating effect for the inflationary bias disappears. then markup is lower because of lower trade costs + if asymmetric markup shocks, bias of MP is lower.

Consider an asymmetric shock. In such a case the bias from markups of the boom country leads to and inflationary bias as $\left(1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right) > 0$ while the recession country induces a deflationary bias s $\left(1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right) < 0$

A.8 Closed form solution of Consumption and Labor

A.8.1 National Currency under Commitment

Plug in monetary policy in a world with national currencies only:

$$C_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}\left(\frac{\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}\right)^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{1-\gamma}}$$

$$\Rightarrow C_t = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w a_t\left(\frac{a_t}{a_t^*}\right)^{\gamma-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}} = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w a_t^{\gamma} a_t^{*(1-\gamma)}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}}$$

Foreign Consumption

$$C_t^* = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w a_t^{1-\gamma} a_t^{*\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa)^{\gamma}}$$

If you plug int both forms of consumption into the exchange rate condition, this equation is true, because the exchange rate is augmented by $(1-\gamma)/\gamma$ labor is given by:

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right)$$

Plugging in monetary policy:

$$\begin{aligned} L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma\frac{\kappa a_t^{-1}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\kappa a_t^{-1}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]}\right) \\ \Rightarrow L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) \end{aligned}$$

A.8.2 National Currency under Discretion

Plug in monetary policy in a world with national currencies only:

$$\begin{aligned} C_t &= \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w\frac{\gamma}{\Theta^N}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}\left(\frac{\frac{\frac{\gamma}{\Theta^N}\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\frac{\gamma}{\Theta^{*N}}\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}\right)^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{1-\gamma}} \\ \Rightarrow C_t &= \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\frac{\gamma}{\Theta^N}a_t\left(\frac{a_t}{a_t^*}\right)^{\gamma-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}} = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\frac{\gamma}{\Theta^N}\frac{a_t^{\gamma}a_t^{*(1-\gamma)}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa)^{(1-\gamma)}} \end{aligned}$$

To keep the expression tractable, I assumed that $\frac{\gamma}{\Theta^N} = \frac{\gamma}{\Theta^* N}$. Foreign Consumption is

$$C_t^* = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\frac{\gamma}{\Theta^N}\frac{a_t^{1-\gamma}a_t^{*\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa)^{*\gamma}}$$

If you plug int both forms of consumption into the exchange rate condition, this equation is true, because the exchange rate is augmented by $(1-\gamma)/\gamma$. Labor is given by:

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right)$$

Plugging in monetary policy:

$$
\begin{aligned}
L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma\frac{\kappa a_t^{-1}\frac{\gamma}{\Theta^N}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\kappa a_t^{-1}\frac{\gamma}{\Theta^N}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]}\right)\\
\Rightarrow L_t(h) &= \frac{1}{\Phi\kappa}\frac{\gamma}{\Theta^N}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right)\\
\Rightarrow L_t^*(f) &= \frac{1}{\Phi^*\kappa}\frac{\gamma}{\Theta^{*N}}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right)
\end{aligned}
$$

A.8.3 National Currency, Commitment in H and Discretion in F

Plug in monetary policy in a world with national currencies only:

$$
\begin{aligned}
C_t &= \big(\frac{1}{1+\varpi}\big)^{1-\gamma}\frac{\gamma_w\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}\left(\frac{\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\frac{\gamma}{\Theta^{*N}}\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}\right)^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{1-\gamma}}\\
\Rightarrow C_t &= \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w a_t\big(\frac{a_t}{a_t^*\frac{\gamma}{\Theta^{*N}}}\big)^{\gamma-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}} = \big(\frac{1}{1+\varpi}\frac{\gamma}{\Theta^{*N}}\big)^{1-\gamma}\gamma_w\frac{a_t^{\gamma}a_t^{*(1-\gamma)}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}}
\end{aligned}
$$

If only one country has a bias, it is transmitted through the exchange rate. Foreign Consumption is

$$
\begin{aligned}
C_t^* &= \big(\frac{1}{1+\varpi}\big)^{1-\gamma}\frac{\gamma_w\frac{\gamma}{\Theta^{*N}}\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}\left(\frac{\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\frac{\gamma}{\Theta^{*N}}\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}\right)^{1-\gamma}}{(\Phi\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{1-\gamma}(\Phi^*\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{\gamma}}\\
\Rightarrow C_t^* &= \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\frac{\gamma}{\Theta^{*N}}a_t\big(\frac{a_t}{a_t^*\frac{\gamma}{\Theta^{*N}}}\big)^{1-\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa^*)^{\gamma}} = \big(\frac{1}{1+\varpi}\big)^{1-\gamma}\gamma_w\left(\frac{\gamma}{\Theta^{*N}}\right)^{\gamma}\frac{a_t^{1-\gamma}a_t^{*\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa^*)^{\gamma}}
\end{aligned}
$$

Both countries end up consuming less of the non-domestic good. If you plug in both forms of consumption into the exchange rate condition, this equation is true, because the exchange rate is augmented by $(1-\gamma)/\gamma$. Labor is given by:

$$
L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right)
$$

Plugging in monetary policy for H

$$
\begin{aligned}
L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma\frac{\kappa a_t^{-1}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\kappa a_t^{-1}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]}\right)\\
\Rightarrow L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right)
\end{aligned}
$$

and for F Plugging in monetary policy:

$$
\begin{aligned}
L_t^*(f) &= \frac{1}{\Phi^*\kappa^*}\left(\gamma\frac{\kappa^* a_t^{*-1}\frac{\gamma}{\Theta^{*N}}\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}{\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*]} + \frac{(1-\gamma)}{1+\varpi}\frac{\kappa^* a_t^{*-1}\frac{\gamma}{\Theta^{*N}}\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}{a_t^{*-1}}}{\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t]}\right)\\
\Rightarrow L_t^*(f) &= \frac{1}{\Phi^*\kappa^*}\frac{\gamma}{\Theta^{*N}}\left(\gamma+\frac{1-\gamma}{1+\varpi}\right)
\end{aligned}
$$

A.8.4 Currency Union

Plug in monetary policy in a world with a currency union.

Now calculate consumption

$$
\begin{aligned}
C_t &= \frac{\gamma_w\left(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\right)^{-1}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}\\
C_t &= \frac{\gamma_w\left(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\right)^{-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{1-\gamma}}
\end{aligned}
$$

The last step only works, if shocks are iid, such that $\mathbb{E}_{t-1}[a_t^{*-1}\mu_t] = \mathbb{E}_{t-1}[a_t^{-1}\mu_t]$. If not, keep it and compute numerically. With $C_t = C_{H,t}^{\gamma}C_{F,t}^{1-\gamma}$, consumption of Home and foreign goods is:

$$
\begin{aligned}
C_{H,t} &= \frac{\gamma\left(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\right)^{-1}}{\Phi\kappa}\\
C_{F,t} &= \frac{(1-\gamma)\left(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\right)^{-1}}{\Phi^*\kappa^*}
\end{aligned}
$$

Labor in a currency union is:

$$
\begin{aligned}
L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma\frac{a_t^{-1}\mu_t}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + (1-\gamma)\frac{a_t^{-1}\mu_t}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right)\\
L_t(h) &= \frac{1}{\Phi\kappa}\left(\frac{a_t^{-1}\left(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\right)^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right)\\
L_t(h) &= \frac{1}{\Phi\kappa}\left(\frac{a_t^{-1}}{\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}}\right)
\end{aligned}
$$

A.9 Allocation and Monetary Policy in Corsetti and Pesenti (2002)

In Corsetti and Pesenti (2002), the consumption basket is symmetric and both countries weight good H with γ:

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma}, \quad C_t^* = C_{H,t}^{*\gamma} C_{F,t}^{*1-\gamma}$$

As a result, the benchmark allocations are different:

Social Planner:

Consumption	$C_{H,t} = \frac{1}{2\kappa} a_t \; C_{H,t}^* = \frac{1}{2\kappa} a_t$
	$C_{F,t} = \frac{1}{2\kappa^*} a_t^* \; C_{F,t}^* = \frac{1}{2\kappa^*} a_t^*$
Labor	$L_t = \frac{1}{\kappa} \; L_t^* = \frac{1}{\kappa^*}$

Flexible Prices (National Currencies)

Consumption	$C_{H,t} = \frac{\gamma}{\kappa} a_t \; C_{H,t}^* = \frac{1-\gamma}{\kappa} a_t$
	$C_{F,t} = \frac{\gamma}{\kappa^*} a_t^* \; C_{F,t}^* = \frac{1-\gamma}{\kappa^*} a_t^*$
Labor	$L_t = \frac{1}{\kappa} \; L_t^* = \frac{1}{\kappa^*}$

Sticky Prices (National Currencies)

Monetary Policy	$\mu_t = \frac{\mathbb{E}[a_t^{-1}\mu_t]}{a_t^{-1}}$
Consumption	$C_t = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma} \gamma a_t^{1-\gamma} a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}} \; C_t^* = \frac{\left(\frac{1}{1+\varpi}\right)^{\gamma} (1-\gamma) a_t^{1-\gamma} a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}}$
Labor	$L_t = \frac{1}{\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) \; L_t^* = \frac{1}{\kappa}\left(\frac{\gamma}{1+\varpi} + (1-\gamma)\right)$

Sticky Prices (Currency Union)

Monetary Policy	$\mu_t = \left(\gamma \frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \frac{(1-\gamma)a_t^{*-1}}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\right)^{-1}$
Consumption	$C_t = \frac{\gamma a_t^{1-\gamma} a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}} \; C_t^* = \frac{(1-\gamma) a_t^{1-\gamma} a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}}$
Labor	$L_t = \frac{1}{\kappa} \frac{a_t^{-1}}{\gamma a_t^{-1} + (1-\gamma) a_t^{*-1}} \; L_t^* = \frac{1}{\kappa} \frac{a_t^{*-1}}{\gamma a_t^{-1} + (1-\gamma) a_t^{*-1}}$

A.10 Consumption, Prices and Labor with Transfers

With transfers from the union-wide planner (superscript P) that benefit country F, consumption of Home agents is lower with transfers: $C_t^P = C_t^U - T_t$. Production needs to satisfy this new demand

$$y_t^P(h) = \left(\frac{\gamma}{\gamma_w}(C_t^U - T_t) + \frac{1-\gamma}{\gamma_w}(C_t^{*U} + T_t)\right)$$
$$y_t^P(f) = \left(\frac{1-\gamma}{\gamma_w}(C_t^U - T_t) + \frac{\gamma}{\gamma_w}(C_t^{*U} + T_t)\right)$$

With transfers going from H to F ($T_t > 0$) overall consumption is shifted from Home goods to foreign goods. As a result, employment in the foreign country increases while it decreases in the Home country, as long as each country has a Home bias ($\gamma > 0.5$).

$$L_t^P = L_t^U + a_t^{-1}\frac{1-2\gamma}{\gamma_w}T_t, \qquad L_t^{*P} = L_t^{*U} - a_t^{*-1}\frac{1-2\gamma}{\gamma_w}T_t$$

There is also an effect on prices, as firms expect the transfer scheme to be in place for the immediate future for most possible states of the world, see also Appendix A.10. In the end, Home firms lower their prices for the next period as the demand for these goods gets lower, while prices of foreign goods increase. The terms of trade (5), defined as prices of foreign exports times the exchange rate over prices of Home exports permanently increase when transfers go to F, see also Figure 12. Recall that with a recession in F and a boom in H, the exchange rate immediately increases with national currencies: As H's monetary policy is optimally more expansive, the exchange rate (Home currency per foreign currency) goes up (H's currency becomes less valuable) and the terms of trade go up as well *permanently*. This might be an unwanted side effect. In the benchmark calibration, the effects are quantitatively very small, as transfers are very small as well. The Euler equation only changes, because of price changes. Lump-sum transfers do not directly distort the intertemporal decision of households. I assume that households do not anticipate the possibility of a 'regime change' in transfers before. That means, if there are zero transfers before, the model is solved as if households do not expect any changes in the transfer scheme before. As soon as the transfers are announced by the social planner, households take the transfers as given and form expectations about it. In the period of announcement, firms adjust their prices for next period taking the future transfers into account. Therefore in the period of transfer announcement, inflation jumps. Note however that this effect is also very small: If transfers go to F from H, prices of F goods rise, while prices of H goods fall. In the aggregate price index these effects partially offset each other. There are only minor effects, for the Foreign country that receives transfers, the aggregate price index goes slightly up, as for F Foreign goods are more important. For H the opposite holds.

Consumption

With transfers from the union-wide planner (superscript P), consumption becomes $C_t^P = C_t^U + T_t$ and $C_t^{*P} = C_t^{*U} - T_t$. The transfers are used by consumers such that the consumption of h goods and f goods changes. The ratio of h goods to the overall consumption bundle is still the same with that specification of preferences. Lump-sum transfers are not distortionary. The ratio is given by:

$$
\begin{aligned}
\frac{C_{H,t}^U}{C_t^U} &= \frac{\frac{\gamma\left(\left(\xi\gamma+(1-\xi)(1-\gamma)\right)a_t^{-1}+\left(\xi(1-\gamma)+(1-\xi)\gamma\right)a_t^{*-1}\right)^{-1}}{\Phi\kappa}}{\left(\frac{\gamma\left(\left(\xi\gamma+(1-\xi)(1-\gamma)\right)a_t^{-1}+\left(\xi(1-\gamma)+(1-\xi)\gamma\right)a_t^{*-1}\right)^{-1}}{\Phi\kappa}\right)^{\gamma}\left(\frac{(1-\gamma)\left(\left(\xi\gamma+(1-\xi)(1-\gamma)\right)a_t^{-1}+\left(\xi(1-\gamma)+(1-\xi)\gamma\right)a_t^{*-1}\right)^{-1}}{\Phi^*\kappa^*}\right)^{1-\gamma}} \\
&= \frac{\gamma}{\gamma_w}
\end{aligned}
$$

Therefore, consumption of h by a Home agent is given by

$$
\begin{aligned}
C_{H,t}^P &= \frac{\gamma}{\gamma_w}(C_t^U - T_t), & C_{H,t}^{*P} &= \frac{1-\gamma}{\gamma_w}(C_t^{*U} + T_t), \\
C_{F,t}^P &= \frac{1-\gamma}{\gamma_w}(C_t^U - T_t), & C_{F,t}^{*P} &= \frac{\gamma}{\gamma_w}(C_t^{*U} + T_t)
\end{aligned}
$$

<u>Prices</u>

Firms know that in a transfer union demand will change. They maximize their profits, accounting for consumer's new demand including transfers. Note that the stocastic discount factor and marginal costs do not change, as lump-sum transfers do not distort the decision of households:

$$
\max_{p_t^P(h),\tilde{p}_t^P(h)} \quad \mathbb{E}_{t-1}[Q_{t-1,t}(((1-\tau)p_t^P(h)-MC_t)\left(\frac{p_t^P(h)}{P_{H,t}}\right)^{-\theta}C_{H,t}^P+((1-\tau)\tilde{p}_t^P(h)-MC_t))\left(\frac{\tilde{p}_t^P(h)}{\tilde{P}_{H,t}}\right)^{-\theta}C_{H,t}^{*P}]
$$

Plug in demand

$$
\begin{aligned}
\max_{p_t^P(h),\tilde{p}_t^P(h)} \quad \mathbb{E}_{t-1}\Big[Q_{t-1,t}\big(((1-\tau)p_t^P(h) - MC_t)\left(\frac{p_t^P(h)}{P_{H,t}}\right)^{-\theta}\frac{\gamma}{\gamma_w}(C_t^U - T_t) \\
+ ((1-\tau)\tilde{p}_t^P(h) - MC_t))\left(\frac{\tilde{p}_t^P(h)}{\tilde{P}_{H,t}}\right)^{-\theta}\frac{1-\gamma}{\gamma_w}(C_t^{*U} + T_t)\big)\Big]
\end{aligned}
$$

Write the problem in state-contingent form, dropping the time index for simplicity:

$$
\begin{aligned}
\max_{p^P(h),\tilde{p}^P(h)} \quad \sum_s p(s \mid s^{-1})\Big[Q_{t-1,t}(s_t)\big(((1-\tau)p^P(h) - MC(s_t))\left(\frac{p^P(h)}{P_H}\right)^{-\theta}\frac{\gamma}{\gamma_w}(C^U(s_t) - T(s^t)) \\
+ ((1-\tau)\tilde{p}(h) - MC(s_t)))\left(\frac{\tilde{p}(h)}{\tilde{P}_H}\right)^{-\theta}\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t) + T(s^t))\big)\Big]
\end{aligned}
$$

The first order condition is with respect to $p^P(h)$

$$\sum_s p(s \mid s^{-1})\Bigg[Q_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\left(\frac{p^P(h)}{P_H}\right)^{-\theta} + MC(s_t)\theta p^P(h)^{-1}\left(\frac{p^P(h)}{P_H}\right)^{-\theta}\right)\cdot \left(\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))\right)\Bigg] = 0$$

Due to symmetric firms we have $P_{H,t} = p_t(h)$. One period price stickiness means that the price $p^P(h)$ is predetermined and does not depend on the state. Therefore, we arrive at

$$p^P(h) = \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s\,|\,s^{-1})\left[Q_{t-1,t}(s_t)\left(MC(s_t)\left(\frac{\gamma}{\gamma_w}\left(C^U(s_t)-T(s^t)\right)\right)\right)\right]}{\sum_s p(s\,|\,s^{-1})\left[Q_{t-1,t}(s_t)\left(\left(\frac{\gamma}{\gamma_w}\left(C^U(s_t)-T(s^t)\right)\right)\right)\right]}$$

Turn to the stochastic discount factor $Q_{t-1,t}(s_t) = \beta\frac{P_{t-1}C_{t-1}(s_t)}{P_tC_t(s_t)}$. Note, that this discount factor was derived from the Euler equation and is not a function of Transfers. The Transfers are lump-sum and do not distort household's intertemporal decision. Therefore, we arrive at

$$p^P(h) = \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s\,|\,s^{-1})\left[(MC(s_t)(\frac{\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))}{(C^U(s_t))}\right]}{\sum_s p(s\,|\,s^{-1})\left[\left(\frac{\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))}{(C^U(s_t))}\right)\right]}$$

With $T(s^t) = 0$, we arrive at the same condition for prices as before. As shown before, Transfers are a constant fraction of GDP, therefore $T(s^t)/C^U(s_t)$ is the same value for all states, except for the state, in which transfers reverse.

$$p^P(h) = \frac{\theta}{(\theta-1)(1-\tau)}\sum_s p(s\,|\,s^{-1})MC(s_t)$$

setting prices equal to expected marginal costs times the markup. NFor the price in the foreign market $\tilde{p}^P(h)$, the firm has the following first order condition

$$\sum_s p(s \mid s^{-1})\Bigg[Q_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\left(\frac{\tilde{p}^P(h))}{\tilde{P}_H^P}\right)^{-\theta} + MC(s_t)\theta \tilde{p}^P(h)^{-1}\left(\frac{\tilde{p}^P(h)}{\tilde{P}_H^P}\right)^{-\theta}\right)\cdot \left(\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Bigg] = 0$$

Rearrange

$$
\begin{aligned}
&-\sum_s p(s\,|\,s^{-1})\Big[Q_{t-1,t}(s_t)\,((1-\tau)(1-\theta))\left(\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big] \\
&=\sum_s p(s\,|\,s^{-1})\Big[Q_{t-1,t}(s_t)\left(MC(s_t)\theta\tilde{p}^P(h)^{-1}\right)\cdot\left(\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big]
\end{aligned}
$$

As $\tilde{p}^P(h)$ is predetermined we can draw it out and arrive at

$$
\begin{aligned}
\tilde{p}^P(h) &= \frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\,|\,s^{-1})\Big[Q_{t-1,t}(s_t)\left(MC(s_t)\left(\frac{1-\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)\right)\right)\Big]}{\sum_s p(s\,|\,s^{-1})\Big[Q_{t-1,t}(s_t)\left(\left(\frac{1-\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)\right)\right)\Big]} \\
&= \frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\,|\,s^{-1})\Big[\left(MC(s_t)\left(\frac{\frac{1-\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)}{C^U(s_t)}\right)\right)\Big]}{\sum_s p(s\,|\,s^{-1})\Big[\left(\frac{\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))}{C^U(s_t)}\right)\Big]}
\end{aligned}
$$

$C^{*U}(s_t)/C^U(s_t)$ are still the same in all states, $T(s^t)/C^U(s_t)$ is also the same in all states, except for the state with a huge asymmetric shock.

$$
\max_{p_t^{*P}(f),p_t(f)}\quad \mathbb{E}_{t-1}\Bigg[Q^*_{t-1,t}(((1-\tau)p_t^{*P}(f)-MC_t^{*P})\left(\frac{p_t^{*P}(f)}{P^*_{F,t}}\right)^{-\theta}C^{*P}_{F,t}+((1-\tau)p_t(f)-MC_t^{*P}))\left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta}C^P_{F,t}
$$

The first order condition with respect to $p_t^{*P}(f)$ is

$$
\begin{aligned}
\sum_s p(s\mid s^{-1})\Big[Q^*_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\left(\frac{p^{*P}(f)}{P^*_F}\right)^{-\theta}+MC(s_t)\theta p^{*P}(f)^{-1}\left(\frac{p^{*P}(f)}{P^*_F}\right)^{-\theta}\right)\cdot \\
\left(\frac{\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big]=0
\end{aligned}
$$

$$
\begin{aligned}
p^{*P}(f) &= \frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\,|\,s^{-1})\Big[Q^*_{t-1,t}(s_t)\left(MC^{*P}(s_t)\left(\frac{\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)\right)\right)\Big]}{\sum_s p(s\,|\,s^{-1})\Big[Q^*_{t-1,t}(s_t)\left(\left(\frac{\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)\right)\right)\Big]} \\
&= \frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\,|\,s^{-1})\Big[\left(MC^{*P}(s_t)\left(\frac{\gamma}{\gamma_w}\frac{\left(C^{*U}(s_t)+T(s^t)\right)}{C^{*U}(s_t)+T(s^t)}\right)\right)\Big]}{\sum_s p(s\,|\,s^{-1})\Big[\left(\left(\frac{\gamma}{\gamma_w}\frac{(C^{*U}(s_t)+T(s^t))}{C^{*U}(s_t)+T(s^t)}\right)\right)\Big]} \\
&= \frac{\theta}{(\theta-1)(1-\tau)}\mathbb{E}[MC_t^{*P}]
\end{aligned}
$$

for foreign good prices in the Home country, we have

$$p^P(f) = \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s\,|\,s^{-1}) \left[Q^*_{t-1,t}(s_t) \left(MC^{*P}(s_t) \left(\frac{1-\gamma}{\gamma_w} \left(C^U(s_t) - T(s^t) \right) \right) \right) \right]}{\sum_s p(s\,|\,s^{-1}) \left[Q^*_{t-1,t}(s_t) \left(\left(\frac{1-\gamma}{\gamma_w} \left(C^U(s_t) - T(s^t) \right) \right) \right) \right]}$$

$$= \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s\,|\,s^{-1}) \left[\left(MC^{*P}(s_t) \left(\frac{1-\gamma}{\gamma_w} \frac{\left(C^U(s_t) - T(s^t) \right)}{C^{*U}(s_t) + T(s^t)} \right) \right) \right]}{\sum_s p(s\,|\,s^{-1}) \left[\left(\left(\frac{1-\gamma}{\gamma_w} \frac{\left(C^U(s_t) - T(s^t) \right)}{C^{*U}(s_t) + T(s^t)} \right) \right) \right]}$$

With that we can calculate the corresponding national price indices under the planner regime

$$P^P_t = \frac{P^{1-\gamma}_{F,t} P^{\gamma}_{H,t}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}, \qquad P^{*P}_t = \frac{P^{*\gamma}_{F,t} P^{*1-\gamma}_{H,t}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}.$$

<u>Labor</u>

Firm stand ready to satisfy demand

$$\begin{aligned} L_t(h) &= a_t^{-1} \left(C^P_{H,t} + C^{*P}_{H,t} \right) \\ &= a_t^{-1} \left(\frac{\gamma}{\gamma_w} (C^U_t - T_t) + \frac{1-\gamma}{\gamma_w} (C^{*U}_t + T_t) \right) \end{aligned}$$

A.11 Nominal Equilibrium

The optimal monetary rules do not pin down the nominal equilibrium. To address this issue, I follow Corsetti and Pesenti (2005) and define two rules $\hat{\mu}_t$ and $\hat{\mu}^*_t$ such that:

$$\hat{\mu}_t = \mu_t \left(\frac{P_{H,t}}{\bar{P}_H} \right)^{\delta}, \quad \hat{\mu}^*_t = \mu^*_t \left(\frac{P^*_{F,t}}{\bar{P}^*_F} \right)^{\delta^*}$$

where $\delta, \delta^* < 0$ are two small negative constants and $\bar{P}_H, \bar{P}^*_F$ are nominal targets of the government. Consider the price of the Home good $P_{H,t}$:

$$\begin{aligned} P_{H,t} &= \Phi \mathbb{E}_{t-1}[\kappa a_t^{-1} \hat{\mu}_t] \\ P_{H,t} &= \Phi \mathbb{E}_{t-1} \left[\kappa a_t^{-1} \mu_t \left(\frac{P_{H,t}}{\bar{P}_H} \right)^{\delta} \right] \\ P_{H,t} &= P_{H,t} \left(\frac{P_{H,t}}{\bar{P}_H} \right)^{\delta} \end{aligned}$$

The solution to that is $P_{H,t} = \bar{P}_H$. Therefore, the governments set an anchor and credibly threatens to adjust monetary policy, if the price deviates from the target. Given the target for domestically produced goods, the prices for imported goods can be computed: Under PCP we have

$$\begin{aligned}
P_{F,t} &= \Phi^* \mathbb{E}[\kappa^* a_t^{*-1} \mu_t^*] \mathcal{E}_t \\
&= \Phi^* \mathbb{E}[\kappa^* a_t^{*-1} \mu_t^*] \frac{\mu_t}{\mu_t^*} \\
&= \Phi^* \mathbb{E}[\kappa^* a_t^{*-1} \mu_t^*] \frac{\frac{\mathbb{E}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\mathbb{E}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}} \\
&= \Phi^* \kappa^* \frac{\frac{\frac{\kappa\Phi}{\kappa\Phi}\mathbb{E}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{1}{a_t^{*-1}}} \\
&= \Phi^* \kappa^* \frac{\frac{\frac{\kappa\Phi}{\kappa\Phi}\mathbb{E}[a_t^{-1}\mu_t \left(\frac{P_{H,t}}{\bar{P}_H}\right)^\delta]}{a_t^{-1}}}{\frac{1}{a_t^{*-1}}} \\
&= \Phi^* \kappa^* \frac{\frac{\frac{1}{\kappa\Phi}\bar{P}_H}{a_t^{-1}}}{\frac{1}{a_t^{*-1}}} \\
P_{F,t} &= \frac{\Phi^*\kappa^*}{\Phi\kappa} \bar{P}_H \frac{a_t^{*-1}}{a_t^{-1}}
\end{aligned}$$

Note that the non-domestic good price fluctuates because of the flexible exchange rate. For the currency union, the central bank just sets the anchor $\bar{P}_H$?

A.12 Problem with Two-Sided Limited Commitment

A.12.1 Functional Equation

Why can the Pareto frontier (19) be described by the recursive problem. The histories of the constraints are potentially large dimensional objects. Thomas and Worrall (1988) show in their work that the problem can indeed be written as a recursive program. The dimensions are contained by using an accounting system cast solely in terms of promised utility. Promised utility is a state variable and summarizes all relevant aspects of an agent's history. With this we can formulate the problem recursively. Expected Lifetime utility for F is rewritten in utility today plus expected lifetime utility in the future. It is a function of promised utility u_s, the state variable of the problem. The constraints are also rewritten in this form. Bellman's principal of optimality states that if a program is optimal in t onwards for state s, it is also optimal in t+1 onwards for all possible states.

A remarkable result is that the appropriate state variable (promised utility) equals future expected utility $u_s = \mathbb{E}_{t-1}[\sum_{j=0}^{\infty} \beta^j u(c_{t+j})]$. Why does promised utility equal the continuation value? Lemma 1 of Thomas and Worrall (1988) states that for each promised value $u_a \in [W^{*N}, W^{*Max}]$ there exists a unique continuation value of the contract δ at time t in which $W(\delta; (h^{t-1}, s_t)) = u_s$ and $W^*(\delta; (h^{t-1}, s_t)) = W^*(u_s)$. The proof is the following: Existence follows from the compactness of all possible future promises. Uniqueness from the convexity of all self-enforcing allocations Γ and the concavity of utility.

Utility in this setup is concave, increasing and continuously differentiable.

Γ is convex: Consider two self-enforcing contracts δ δ' that promise a sequence of consumption $\{C_t(\delta, s^t)\}_{t=0}^{\infty}, \{C_t(\delta', s^t)\}_{t=0}^{\infty}$. Let the convex combinations of both contracts be denoted by δ^λ with consumption streams $\{C_t(\delta^\lambda.s^t)\}_{t=0}^{\infty}$ By the concavity of utility, it holds that: $W(\delta^\lambda, s^t) \geq \lambda W(\delta, s^t) + (1-\lambda)W(\delta', s^t)$. Therefore, the convex combination of both sustainable contracts is sustainable as well.

Promised utility is compact: Proof for that: Let I_s b the set of feasible values of u_s. If $u_s \in I_s$, then $u'_s \in I_s \forall u'_s \in [W^N, u_s)$ is I_s closed? Consider a sequence $u_s^\nu \in I_s$ such that $\lim_{\nu\to\infty} u_s^\nu = u_s$ with a corresponding consumption stream (contract δ^ν). For a given parameterization, consumption is contained in an interval, say $[a, b]$, therefore the contract specifies only a countable number of consumption streams, the space of contracts is sequentially compact on the product topology. So, there is a sub-sequence of contracts converging pointwise to a limiting contract δ^∞. Since utility V is continuous and $\beta \in (0, 1)$, by the dominated convergence theorem after any history the limit of the gain to an agent equals the gain from the limiting contract, for both agents. Therefore δ^∞ is self-enforcing since each δ^ν is and gives promise utility of u_s.

A.12.2 Consumption and Continuation Values

Let $\bar{C}_{s_2} \equiv \bar{C}_{Hs_2}^{\gamma}\bar{C}_{Fs_2}^{1-\gamma}$ denote consumption of agent H if F's participation constraint binds (in state s_2). In that case H receives continuation value $\bar{u}_{s_2}^-$. $\underline{C}_{s_1} \equiv \underline{C}_{Hs_1}^{\gamma}\underline{C}_{Fs_1}^{1-\gamma}$ is H's consumption if H's participation constraint binds (in state s_1) in which H receives continuation value $\bar{u}_{s_1}^+$. Introduce the same notation for every state. The ergodic consumption set consists of

$$\left\{[\bar{C}_{s_2}, \underline{C}_{s_1}] \cap \{\bar{C}_{s_2}, \bar{C}_{s_3}, \underline{C}_{s_3}, \bar{C}_{s_4}, \underline{C}_{s_4}, \underline{C}_{s_1}\}\right\}$$

The set consist at least out of two points $\bar{C}_{s_2}, \underline{C}_{s_1}$ and at most out of four additional ones. When there are no first best sustainable allocations, participation constraints of H and F bind in state s_1 and s_2. Consider the state s_1. H's participation constraint is:

$$V(\underline{C}_{s_1}) + \beta\bar{u}_{s_1}^+ = V^N(C_{s_1}^N) + \beta W^N$$

F gets $Y_{s_1} - \underline{C}_{s_1}$ and gets continuation value $W^*(\bar{u}_{s_1}^+) \equiv \bar{u}_{s_1}^-$. The consumption allocation in state s_3 depends on different promised continuation values with which agents enter the period. Let $\hat{C}_{s_3}$ denote consumption of H in state s_3 when H was at the participation constraint before. Let $\hat{u}_{s_3}^+$ denote the continuation value in state 3 if the participation constraint was binding before. As H was at the participation constraint before, consumption and promised utility were increased in state 1, after that in state 3 is "payback time", because agents in H were promised a higher continuation value. Agent F gets $Y_{s_3} - \hat{C}_{s_3}$

and continuation value $W^*(\hat{u}^+_{s_3})$. The participation constraint might bind :

$$\begin{aligned} V^{*U}(\hat{C}^*_{s_3}) + \beta W^*(\hat{u}^+_{s_3}) &\geq V^{*N}(C^{*N}_{s_3}) + \beta W^{*N} \\ V(\hat{C}_{s_3}) + \beta \hat{u}^+_{s_3} &\geq V^N(C^N_{s_3}) + \beta W^N \end{aligned}$$

Foreign consumption is just a function of production minus Home consumption. The continuation value can be explicitly written:

$$\begin{aligned} \hat{u}^+_{s_4} = \hat{u}^+_{s_3} = \bar{u}^+_{s_1} = \quad & p^2(V(\hat{C}_{s_3}) + \beta\hat{u}^+_{s_3}) + p(1-p)(V(\bar{C}_{s_2}) + \beta\bar{u}^-_{s_2}) \\ & +(1-p)^2(V(\hat{C}_{s_4}) + \beta\hat{u}^+_{s_4}) + (1-p)p(V(\underline{C}_{s_1}) + \beta\bar{u}^+_{s_1}) \\ \hat{u}^-_{s_4} = \hat{u}^-_{s_3} = \bar{u}^-_{s_2} = \quad & p^2(V(Y_{s_3} - \hat{C}_{s_3}) + \beta\hat{u}^-_{s_3}) + p(1-p)(V(\bar{C}_{s_2}) + \beta\bar{u}^-_{s_2}) \\ & +(1-p)^2(V(Y_{s_4} - \hat{C}_{s_4}) + \beta\hat{u}^-_{s_4}) + (1-p)p(V(\underline{C}_{s_1}) + \beta\bar{u}^+_{s_1}) \end{aligned}$$

Where the equalities of u^+ and u^- follow from the optimality condition $W^{*'}(u_s) = W^{*'}(u_0)$, meaning that in a state where no PC binds, promised utility is unchanged. Therefore, there are at most two distinct continuation values.

$$\begin{aligned} u^+ &= p^2\big(V(\hat{C}_{s_3}) + \beta u^+\big) + p(1-p)\big(V(\bar{C}_{s_2}) + \beta u^-\big) \\ &\quad +(1-p)^2\big(V(\hat{C}_{s_4}) + \beta u^+\big) + (1-p)p\big(V(\underline{C}_{s_1}) + \beta u^+\big) \\ \Rightarrow u^+\Bigg(1-\beta\big(p^2+(1-p)^2+(1-p)p\big)\Bigg) &= p^2V(\hat{C}_{s_3}) + p(1-p)(V(\bar{C}_{s_2}) + \beta u^-) \\ &\quad +(1-p)^2V(\hat{C}_{s_4}) + (1-p)pV(\underline{C}_{s_1}) \\ \Rightarrow u^+ = \frac{1}{1-\beta\big(p^2+(1-p)^2+(1-p)p} &\Big[p^2V(\hat{C}_{s_3}) + p(1-p)(V(\bar{C}_{s_2}) + \beta u^-) \\ &\quad +(1-p)^2V(\hat{C}_{s_4}) + (1-p)pV(\underline{C}_{s_1})\Big] \end{aligned}$$

And the continuation value of the agent who is not at the PC:

$$\begin{aligned} u^- &= p^2(V(Y_{s_3} - \hat{C}_{s_3}) + \beta u^-) + p(1-p)(V(\bar{C}_{s_2}) + \beta u^-) \\ &\quad +(1-p)^2(V(Y_{s_4} - \hat{C}_{s_4}) + \beta u^-) + (1-p)p(V(\underline{C}_{s_1}) + \beta u^+) \\ \Rightarrow u^- = \frac{1}{1-\beta\big(p^2+(1-p)^2+(1-p)p} &\Big[p^2V(Y_{s_3} - \hat{C}_{s_3}) + p(1-p)V(\bar{C}_{s_2}) \\ &\quad +(1-p)^2V(Y_{s_4} - \hat{C}_{s_4}) + (1-p)p\big(V(\underline{C}_{s_1}) + \beta u^+\big)\Big] \end{aligned}$$

Now plug in u^+ in u^- and solve for it:

$$u^- = \frac{1}{1-\beta\big(p^2+(1-p)^2+(1-p)p}\Big[p^2V(Y_{s_3}-\hat{C}_{s_3})+p(1-p)(V(\bar{C}_{s_2})$$
$$+(1-p)^2V(Y_{s_4}-\hat{C}_{s_4})+(1-p)p\bigg(V(\underline{C}_{s_1})$$
$$+\beta\bigg(\frac{1}{1-\beta\big(p^2+(1-p)^2+(1-p)p}\Big[p^2V(\hat{C}_{s_3})+p(1-p)\big(V(\bar{C}_{s_2})+\beta u^-\big)$$
$$+(1-p)^2V(\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})\Big]\bigg)\bigg)\Big]$$

Let the fraction in front of the terms be denoted by $D = \frac{1}{1-\beta\big(p^2+(1-p)^2+(1-p)p}$. Therefore

$$u^- = D\Big[p^2V(Y_{s_3}-\hat{C}_{s_3})+p(1-p)(V(\bar{C}_{s_2})$$
$$+(1-p)^2V(Y_{s_4}-\hat{C}_{s_4})+(1-p)p\bigg(V(\underline{C}_{s_1})$$
$$+\beta\bigg(D\Big[p^2V(\hat{C}_{s_3})+p(1-p)\big(V(\bar{C}_{s_2})+\beta u^-\big)$$
$$+(1-p)^2V(\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})\Big]\bigg)\bigg)\Big]$$

Isolating u^-:

$$u^- - D(1-p)p\beta Dp(1-p)\big(\beta u^-\big) = D\Big[p^2V(Y_{s_3}-\hat{C}_{s_3})+p(1-p)(V(\bar{C}_{s_2})$$
$$+(1-p)^2V(Y_{s_4}-\hat{C}_{s_4})+(1-p)p\bigg(V(\underline{C}_{s_1})$$
$$+\beta\bigg(D\Big[p^2V(\hat{C}_{s_3})+p(1-p)V(\bar{C}_{s_2})$$
$$+(1-p)^2V(\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})\Big]\bigg)\bigg)\Big]$$

now

$$u^- = \frac{D}{1-(D(1-p)p\beta)^2}\Big[p^2V(Y_{s_3}-\hat{C}_{s_3})+p(1-p)V(\bar{C}_{s_2})+(1-p)^2V(Y_{s_4}-\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})$$
$$+(1-p)p\beta\bigg(D\Big[p^2V(\hat{C}_{s_3})+p(1-p)V(\bar{C}_{s_2})+(1-p)^2V(\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})\Big]\bigg)\Big]$$

For u^+:

$$u^- = \frac{D}{1-(D(1-p)p\beta)^2}\Big[p^2V(\hat{C}_{s_3})+p(1-p)V(\bar{C}_{s_2})+(1-p)^2V(\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})$$
$$+(1-p)p\beta\Big(D\Big[p^2V(Y_{s_3}-\hat{C}_{s_3})+p(1-p)V(\bar{C}_{s_2})+(1-p)^2V(Y_{s_4}-\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})\Big]\Big)\Big]$$

One solution in a symmetric 4 state example is: Both continuation values are the same. The central bank always promises the same future utility under the new monetary regime. In that case consumption in a symmetric state is always the same for both ($\hat{C}_s = Y - \hat{C}_s$))). As in Ljungqvist and Sargent (2004), unchanged continuation values lay on the Pareto frontier if shocks are iid and the consumption intervals do not overlap. The only thing, that changes in a period is consumption, as there is no efficient way to deliver a change in continuation values. Note that the Pareto Frontier outlined in **??** is non-differentiable at the points $(u^+, u^-) = (u, W^*(u))$ and $(u^-, u^+) = (u, W^*(u))$.
For $\hat{C}_s = Y - \hat{C}_s$)) expected lifetime utility appears twice in the square brackets of continuation values. Therefore, they can be drawn out, using the third binomial formula on the denominator in front and then $1 + Dp(1-p)\beta$ cancels out. The continuation value is

$$u^+ = u^- = u = \frac{D}{1-Dp(1-p)\beta}\Big[p^2V(\hat{C}_{s_3})+p(1-p)V(\bar{C}_{s_2})+(1-p)^2V(\hat{C}_{s_4})+(1-p)pV(\underline{C}_{s_1})\Big]$$

From the promise keeping constraint, we can calculate the continuation value. As $u_0 = u_s = u$

$$\sum_{s\in A} p_s(V(C_s^U, L_s^U) + \beta u) = u$$
$$\frac{1}{1-\beta}\sum_{s\in A} p_s(V(C_s^U, L_s^U)) = u$$

which means that the continuation value of the social planner is just expected lifetime utility in a monetary union under the new policy rule. This can be computed and is just a number as is the outside option of lifetime utility with national currencies. This pins down the consumption distribution for all states. For the symmetric states we have:

$$\frac{1-\gamma}{Y_{Hs_1} - C_{Hs_1}} / \frac{\gamma}{C_{Hs_1}} = -W'(u_{s_1}) = 1$$

As all continuation values are the same, that derivative is just 1. Consumption of good

H in that state is

$$C_{Hs_3} = \gamma Y_{Hs_3}, \quad C^*_{Hs_3} = (1-\gamma) Y_{Hs_3}$$

which is the same allocation of the social planner and of the central bank with the ordinary policy rule in that state. Consumption of the other good and of the foreign household are given by:

$$C_{Fs_3} = (1-\gamma) Y_{Fs_3}, \quad C^*_{Fs_3} = \gamma Y_{Fs_3}$$

Some for the symmetric state s_4.
Consider state s_1 in which H's participation constraint is binding. In that case consumption for H is pinned down by:

$$\begin{aligned} \ln(C_{s_1}) - \kappa L_{s_1} &= V^N_{s_1} + \beta W^N - \beta u \\ C_{s_1} &= \exp(V^N_{s_1} + \beta W^N - \beta u + \kappa L_{s_1}) \end{aligned}$$

As the elasticity of substitution is one, both kind of goods increase/decrease 1 to 1 in a changed allocation. F gets $Y_{s_1} - C_{s_1}$. This allocation is sustainable, if PC for F is not violated. In state s_2 the allocation is mirrored. Now we can trace out lifetime utility in the 4-period example. Consumption could be plotted as well!

A.12.3 A Specific Example

Focus on a specific example to determine the path of consumption and promised utility. In the initial period the Home country is at its participation constraint, this is state $s_1 = \{a^r, a^b\}$. Output is $Y_{Hs_1} = L_{s_1} a^r$ and $Y_{Fs_1} = L^*_{s_1} a^b$. Let $\utilde{C}_{s_1} \equiv \utilde{C}^{\gamma}_{Hs_1} \utilde{C}^{1-\gamma}_{Fs_1}$ denote consumption that is awarded to agent H if H's participation constraint binds in that state. H receives continuation value $\bar{u}^+_{s_1}$. For state s_2 let $\bar{C}_{s_2} \equiv \bar{C}^{\gamma}_{Hs_2} \bar{C}^{1-\gamma}_{Fs_2}$ be H's consumption if F's participation constraint binds in which H receives continuation value $\bar{u}^-_{s_2}$.

H's participation constraint in s_1 is:

$$V(\utilde{C}_{s_1}) + \beta \bar{u}^+_{s_1} = V^N(C^N_{s_1}) + \beta W^N$$

F gets $Y_{s_1} - \utilde{C}_{s_1}$ and gets continuation value $W^*(\bar{u}^+_{s_1}) \equiv \bar{u}^-_{s_1}$. How is the continuation value determined? Remember that the continuation value consists of consumption and future continuation values. Therefore, consider consumption and continuation values of all states in the next period. For a binding participation constraints in s_1 and s_2 the Home agents would receive $\utilde{C}_{s_1}, \bar{u}^+_{s_1}$ and $\bar{C}_{s_2}, \bar{u}^-_{s_2}$ respectively in the next period. For the symmetric states s_3 and s_4 in which no PC binds, consumption and promised utility depend on the promised utility with which agents enter the period. Let $\hat{C}_{s_3}$ denote consumption of H in state s_3 when H was at the participation constraint before. Let

$\hat{u}_{s_3}^{+}$ denote the continuation value in that case. As H was at the participation constraint before, consumption and promised utility were increased in state 1, after that in state 3 is "payback time", because agents in H were promised a higher continuation value. Agent F gets $Y_{s_3} - \hat{C}_{s_3}$ and continuation value $W^*(\hat{u}_{s_3}^{+}) \equiv \hat{u}_{s_3}^{-}$. Participation constraints must be satisfied:

$$V^{*U}(\hat{C}_{s_3}^{*}) + \beta W^*(\hat{u}_{s_3}^{+}) \geq V^{*N}(C_{s_3}^{*N}) + \beta W^{*N}$$
$$V(\hat{C}_{s_3}) + \beta \hat{u}_{s_3}^{+} \geq V^N(C_{s_3}^N) + \beta W^N$$

The same notation can be introduced for state s_4 in the next period. Therefore, in the initial state s_1 the continuation value for agent H can be written as:

$$\begin{aligned} \bar{u}_{s_1}^{+} = \quad & p^2(V(\hat{C}_{s_3}) + \beta\hat{u}_{s_3}^{+}) + p(1-p)(V(\bar{C}_{s_2}) + \beta\bar{u}_{s_2}^{-}) \\ & +(1-p)^2(V(\hat{C}_{s_4}) + \beta\hat{u}_{s_4}^{+}) + (1-p)p(V(\underline{C}_{s_1}) + \beta\bar{u}_{s_1}^{+}) \end{aligned}$$

and in state s_2

$$\begin{aligned} \bar{u}_{s_2}^{-} = \quad & p^2(V(\hat{C}_{s_3}) + \beta\hat{u}_{s_3}^{-}) + p(1-p)(V(\bar{C}_{s_2}) + \beta\bar{u}_{s_2}^{-}) \\ & +(1-p)^2(V(\hat{C}_{s_4}) + \beta\hat{u}_{s_4}^{-}) + (1-p)p(V(\underline{C}_{s_1}) + \beta\bar{u}_{s_1}^{+}) \end{aligned}$$

The optimality condition $W^{*'}(u_s) = W^{*'}(u_0)$ states that continuation utility is unchanged if the PC does not bind. Therefore denote $\hat{u}_{s_4}^{+} = \hat{u}_{s_3}^{+} = \bar{u}_{s_1}^{+} \equiv u^+$ and $\hat{u}_{s_4}^{-} = \hat{u}_{s_3}^{-} = \bar{u}_{s_2}^{-} \equiv u^-$. Promised utility u^+ and u^- can hence be written as a function of consumption:

$$\begin{aligned} u^- = \frac{1}{1-(D(1-p)p\beta)^2} D\Bigg[& p^2V(\hat{C}_{s_3}) + p(1-p)(V(\bar{C}_{s_2}) + (1-p)^2V(\hat{C}_{s_4}) \\ & +(1-p)p\Bigg(V(\underline{C}_{s_1}) + \beta\Bigg(D\Bigg[p^2V(\hat{C}_{s_3}) + p(1-p)V(\bar{C}_{s_2}) \\ & +(1-p)^2V(\hat{C}_{s_4}) + (1-p)pV(\underline{C}_{s_1})\Bigg]\Bigg)\Bigg)\Bigg] \end{aligned}$$

where $D = \frac{1}{1-\beta\left(p^2+(1-p)^2+(1-p)p\right)}$

$$\begin{aligned} u^+ = D\Bigg[& p^2V(\hat{C}_{s_3}) + (1-p)^2V(\hat{C}_{s_4}) + (1-p)pV(\underline{C}_{s_1}) + p(1-p)\Bigg(V(\bar{C}_{s_2}) \\ & +\beta\Bigg[\frac{1}{1-(D(1-p)p\beta)^2} D\Bigg[p^2V(\hat{C}_{s_3}) + p(1-p)(V(\bar{C}_{s_2}) + (1-p)^2V(\hat{C}_{s_4}) \\ & +(1-p)p\Bigg(V(\underline{C}_{s_1}) + \beta\Bigg(D\Bigg[p^2V(\hat{C}_{s_3}) + p(1-p)V(\bar{C}_{s_2}) \\ & +(1-p)^2V(\hat{C}_{s_4}) + (1-p)pV(\underline{C}_{s_1})\Bigg]\Bigg)\Bigg)\Bigg]\Bigg]\Bigg)\Bigg] \end{aligned}$$

As all continuation values are the expected value of the monetary union.

The planner still optimizes F's lifetime utility. The relevant objective function is:

$$
\begin{aligned}
\max_{\underline{C}_{Hs_1},\underline{C}_{Fs_1},\bar{C}_{Hs_2},\bar{C}_{Fs_2}\hat{C}_{Hs_3}\hat{C}_{Fs_3}\hat{C}_{Hs_4}\hat{C}_{Fs_4}} \quad & \log\left((Y_{Hs_1}-\underline{C}_{Hs_1})^{1-\gamma}(Y_{Fs_1}-\underline{C}_{Fs_1})^{\gamma}\right)+\beta u^{-} \\
s.t. \log\left((\underline{C}^{\gamma}_{Hs_1})(\underline{C}_{Fs_1})^{1-\gamma}\right)-\kappa l^{U}_{s_1}+\beta u^{+} &= V^{N}_{s_1}+\beta W^{N} \\
\log\left((Y_{Hs_1}-\underline{C}_{Hs_1})^{1-\gamma}(Y_{Fs_1}-\underline{C}_{Fs_1})^{\gamma}\right)-\kappa^{*} l^{*U}_{s_1}+\beta u^{-} &\geq V^{N}_{s_1}+\beta W^{N} \\
\log\left((\bar{C}^{\gamma}_{Hs_2})(\bar{C}_{Fs_2})^{1-\gamma}\right)-\kappa l^{U}_{s_2}+\beta u^{-} &\geq V^{N}_{s_2}+\beta W^{N} \\
\log\left((Y_{Hs_2}-\bar{C}_{Hs_2})^{1-\gamma}(Y_{Fs_2}-\bar{C}_{Fs_2})^{\gamma}\right)-\kappa^{*} l^{*U}_{s_2}+\beta u^{+} &= V^{N}_{s_2}+\beta W^{N} \\
\log\left((Y_{Hs_3}-\hat{C}_{Hs_3})^{1-\gamma}(Y_{Fs_3}-\hat{C}_{Fs_3})^{\gamma}\right)-\kappa^{*} l^{*U}_{s_3}+\beta u^{-} &\geq V^{N}_{s_3}+\beta W^{N} \\
\log\left((\hat{C}^{\gamma}_{Hs_3})(\hat{C}_{Fs_3})^{1-\gamma}\right)-\kappa l^{U}_{s_3}+\beta u^{+} &\geq V^{N}_{s_3}+\beta W^{N}
\end{aligned}
$$

where u^{+} and u^{-} are given above. A useful property in this symmetric setup is, that $Y_{s_2}-\bar{C}_{s_2}=\underline{C}_{A_1}$. Therefore, if the planner satisfies H's PC with $\bar{C}_{s_1}$, so is F's PC in state s_2. For both symmetric states, the PC does not bind (also with the new consumption rule? check MY feeling: consumption between those states is way too different! Other idea as in Ligon et al. (2002): The ratio of marginal rates of substitution stays the same! This way also with changing aggregates, the stuff remains equal!). Therefore, the Lagrangian is given by:

$$
\begin{aligned}
\max_{\underline{C}_{Hs_1},\underline{C}_{Fs_1},\hat{C}_{Hs_3},\hat{C}_{Fs_3}} L = & \log\left((Y_{Hs_1}-\underline{C}_{Hs_1})^{1-\gamma}(Y_{Fs_1}-\underline{C}_{Fs_1})^{\gamma}\right)+\beta u^{-} \\
& +\Lambda_{s_1}\left(\log\left((\underline{C}^{\gamma}_{Hs_1})(\underline{C}_{Fs_1})^{1-\gamma}\right)-\kappa l^{U}_{s_1}+\beta u^{+}-V^{N}_{s_1}-\beta W^{N}\right) \\
& +\Lambda_{s_3}\left(\log\left(\hat{C}_{Hs_3})^{\gamma}(\hat{C}_{Fs_3})^{1-\gamma}\right)-\kappa^{*} l^{*U}_{s_3}+\beta u^{+}-V^{N}_{s_3}-\beta W^{N}\right).
\end{aligned}
$$

The continuation values are given as before. Their derivative can be calculated.

$$
\begin{aligned}
u^{-}_{\underline{C}_{Hs_1}} &= \tfrac{1}{1-(D(1-p)p\beta)^2}D\left[(1-p)pV'+(1-p)p\beta D(1-p)pV'\right] \\
u^{+}_{\underline{C}_{Hs_1}} &= D\left[(1-p)pV'+p(1-p)\beta u^{-}_{\underline{C}_{Hs_1}}\right] \\
u^{-}_{\bar{C}_{Hs_2}} &= \tfrac{1}{1-(D(1-p)p\beta)^2}D\left[p(1-p)V'_{\bar{C}_{Hs_2}}+p(1-p)\beta Dp(1-p)V'_{\bar{C}_{Hs_2}}\right] \\
u^{+}_{\bar{C}_{Hs_2}} &= Dp(1-p)\left[V'+\beta u^{-}_{\bar{C}_{Hs_2}}\right] \\
u^{-}_{\hat{C}_{Hs_3}} &= \tfrac{1}{1-(D(1-p)p\beta)^2}D\left[p^2V'_{\hat{C}_{Hs_3}}+(1-p)p\beta Dp^2V'_{\hat{C}_{Hs_3}}\right] \\
u^{+}_{\hat{C}_{Hs_3}} &= D\left[p^2V'+p(1-p)\beta u^{-}_{\hat{C}_{Hs_3}}\right] \\
u^{-}_{\hat{C}_{Hs_4}} &= \tfrac{1}{1-(D(1-p)p\beta)^2}D\left[(1-p)^2V'_{\hat{C}_{Hs_4}}+(1-p)p\beta D(1-p)^2V'_{\hat{C}_{Hs_4}}\right] \\
u^{+}_{\hat{C}_{Hs_4}} &= D\left[(1-p)^2V'+p(1-p)\beta u^{-}_{\hat{C}_{Hs_4}}\right]
\end{aligned}
$$

Consumption foc is:

$$\frac{\partial L}{\partial \underline{C}_{Hs_1}}: V^{*\prime} + \beta u^{-}_{\underline{C}_{Hs_1}} + \Lambda_{s_1}(V' + \beta u^{+}_{\underline{C}_{Hs_1}}) + \Lambda_{s_2}(\beta u^{+}_{\underline{C}_{Hs_1}}) = 0$$
$$\frac{\partial L}{\partial \bar{C}_{Hs_2}}: \beta u^{-}_{\bar{C}_{Hs_2}} + \Lambda_{s_1}(\beta u^{+}_{\bar{C}_{Hs_2}}) + \Lambda_{s_2}(V^{*\prime} + \beta u^{+}_{\bar{C}_{Hs_2}}) = 0$$
$$\frac{\partial L}{\partial \hat{C}_{Hs_3}}: \beta u^{-}_{\hat{C}_{Hs_3}} + \Lambda_{s_1}(\beta u^{+}_{\hat{C}_{Hs_3}}) + \Lambda_{s_2}(\beta u^{-}_{\hat{C}_{Hs_3}}) = 0$$
$$\frac{\partial L}{\partial \hat{C}_{Hs_4}}: \beta u^{-}_{\hat{C}_{Hs_4}} + \Lambda_{s_1}(\beta u^{+}_{\hat{C}_{Hs_4}}) + \Lambda_{s_2}(\beta u^{-}_{\hat{C}_{Hs_4}}) = 0$$
$$\log\left((\underline{C}^{\gamma}_{Hs_1})(\underline{C}_{Fs_1})^{1-\gamma}\right) - \kappa l^{U}_{s_1} + \beta u^{+} = V^{N}_{s_1} + \beta W^{N}$$
$$\log\left((Y_{Hs_2} - \bar{C}_{Hs_2})^{1-\gamma}(Y_{Fs_2} - \bar{C}_{Fs_2})^{\gamma}\right) - \kappa^{*} l^{*U}_{s_2} + \beta u^{+} = V^{N}_{s_2} + \beta W^{N}$$

A.12.4 Convexity of Sets

Consider a convex set of two sustainable contracts $T(\cdot)$ and $\hat{T}(\cdot)$ after history h^t: $\alpha T(h^t) + (1-\alpha)\hat{T}(h^t)$. By concavity of u and v the average of these two sustainable contracts offers at least as much as the average surplus and is hence also sustainable.

Consider any pair of two sustainable discounted surpluses and consider a convex combination as above. As in Ligon et al. (2002) any discounted surplus between these two surpluses is sustainable as well. Therefore the set of sustainable surpluses is an interval $[\underline{U}_s, \bar{U}_s]$ for H and for F $[\underline{V}_s, \bar{V}_s]$. By definition $\underline{U}_s = 0$, as the minimum surplus of the union cannot be smaller than zero. [22]

A.12.5 Pareto Frontier is concave

Take any two sustainable surpluses $U(s^t)$ and $\hat{U}_s$. Following the same argument as made above, the convex combination $\alpha U_S + (1-\alpha)\hat{U}_s$ will offer H more than the average of these contracts and household F strictly more than the average of the original surpluses, because v is concave. Therefore each V_s is concave.

$V_s(U(s^t))$ is strictly decreasing in $U(s^t)$ on the whole interval $[\underline{U}_s, \bar{U}_s]$, since starting from any $U(s^t) > \underline{U}_s$, there must be some history h_t, such that $U_t(h_t) > 0$. A small increase in $T(s^t)$ leads to an increase in F's utility and a decrease in H's, while not violating the participation constraint. It follows that $U(s^{t+1}) \leq \bar{U}_r$ can be written as $V(s^{t+1}, U(s^{t+1})) \geq \underline{V}_r$.

[22] Non-negativity of consumption must be ensured as well, this will not be a concern in this calibration.

A.12.6 The Lagrangian of the Dynamic Problem

The Lagrangian of the problem (19) is

$$\mathcal{L} = \max_{T(s^t),(U(s^{t+1}))_{r=1}^{S}} \ln\left(C^{*U}(s_t)+T(s^t)\right) - \kappa^* l^{*U}(s_t) - v^N(s_t) + \beta\sum_{r=1}^{S} p(s^{t+1}\,|\,s^t)V(s^{t+1},U(s^{t+1}))$$

$$+ \lambda(s^t)\left[\ln\left(C^U(s_t)-T(s^t)\right) - \kappa l^U(s_t) - u^N(s_t) + \beta\sum_{r=1}^{S} p(s^{t+1}\,|\,s^t)U(s^{t+1}) \geq U(s^t)\right]$$

$$+ \beta p(s^{t+1}\,|\,s^t)\phi(s^{t+1})U_r + \beta p(s^{t+1}\,|\,s^t)\zeta(s^{t+1})V(s^{t+1},U(s^{t+1}))$$

The first order conditions are

$$\mathcal{L}_{T(s^t)}: \quad v'_{T(s^t)} - \lambda u'_{T(s^t)} = 0$$

$$\Rightarrow \frac{u'}{v'} = \lambda \qquad \frac{1}{C^{*U}(s_t)+T(s^t)} - \lambda\frac{1}{C^U(s_t)-T(s^t)} = 0$$

$$\mathcal{L}_{U(s^{t+1})}: \quad \beta p(s^{t+1}\,|\,s^t)V'_r(U(s^{t+1})) + \lambda\beta p(s^{t+1}\,|\,s^t) + \beta p(s^{t+1}\,|\,s^t)\phi(s^{t+1}) + \beta p(s^{t+1}\,|\,s^t)V'_r(U($$

$$\Rightarrow \frac{\lambda(s^t)+\phi(s^{t+1})}{1+\zeta(s^{t+1})} = -V'_r(U(s^{t+1}))$$

Envelope Condition $\quad \lambda(s^t) = V_s(U(s^t))$

A.12.7 Intuition for Transfers

Note that when no new participation constraint binds, (24) tells us, that transfers are given by:

$$T(s^{t+1}) = \frac{C^U(s_{t+1}) - \lambda(s^t)C^{*U}(s_{t+1})}{1+\lambda(s^t)}$$

Recall the perfect risk sharing property of the model, which tells us, that consumption of the Home and the foreign country in the Union are always the same. This means that as soon as the economy is in a synchronized boom (or a synchronized recession), both consumption values simultaneously increase (or decrease) by the same amount. Therefore transfers, that were obtained with the help of (24) increase (or decrease in a recession) by the same amount as consumption does. Therefore, transfers relative to GDP stay always the same, as long as no new participation constraint binds.

A.12.8 Differentiability of the Pareto Frontier

see Koeppl (2004)

A.13 Monetary Policy under two-sided limited Commitment

A.14 Interest rates and Announcements

$$
\begin{aligned}
V_s(U(s^t)) = & \max_{\mu(s^t),(U(s^{t+1}))_{r=1}^S} \ln\left(C^{*U}(\mu(s^t))\right) - \kappa^* l^{*U}(\mu(s^t)) - v^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}\,|\,s^t) V(s^{t+1}\\
\text{s.t.}\ \ [\lambda(s^t)] \ \ & \ln\left(C^U(\mu(s^t))\right) - \kappa l^U(\mu(s^t)) - u^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}\,|\,s^t) U(s^{t+1}) \geq U(s^t)\\
[\beta p(s^{t+1}\,|\,s^t)\phi(s^{t+1})] \ \ & U(s^{t+1}) \geq 0\\
[\beta p(s^{t+1}\,|\,s^t)\zeta(s^{t+1})] \ \ & V(s^{t+1}, U(s^{t+1})) \geq 0\\
& C(s_t) = C_H^{\gamma}(s_t) C_F^{1-\gamma}(s_t)\\
& l(\mu(s^t)) a_s = C_H(\mu(s^t)) + C_H^*(\mu(s^t))\\
& l(\mu(s^t)) a_s = C_F(\mu(s^t)) + C_F^*(\mu(s^t))
\end{aligned}
$$

The Lagrangian is

$$
\begin{aligned}
\mathcal{L} = & \max_{\mu(s^t),(U(s^{t+1}))_{r=1}^S} \ln\left(C^{*U}(\mu(s^t))\right) - \kappa^* l^{*U}(\mu(s^t)) - v^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}\,|\,s^t) V(s^{t+1}, U(s^{t+1}))\\
& + \lambda(s^t)\left(\ln\left(C^U(\mu(s^t))\right) - \kappa l^U(\mu(s^t)) - u^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}\,|\,s^t) U(s^{t+1}) - U(s^t)\right)\\
& + \beta p(s^{t+1}\,|\,s^t)\phi(s^{t+1}) U(s^{t+1}) + \beta p(s^{t+1}\,|\,s^t)\zeta(s^{t+1}) V(s^{t+1}, U(s^{t+1}))
\end{aligned}
$$

The first order condition with respect to the monetary stance today $\mu(s^t)$ is given by

$$
\begin{aligned}
\mathcal{L}'_{\mu(s^t)}: \quad & \frac{C^{*U'}(\mu(s^t))}{C^{*U}(\mu(s^t))} - \kappa^* l^{*U'}(\mu(s^t)) + \lambda(s^t)\left(\frac{C^{U'}(\mu(s^t))}{C^U(\mu(s^t))} - \kappa l^{U'}(\mu(s^t))\right) = 0\\
& \Rightarrow -\frac{\frac{C^{*U'}(\mu(s^t))}{C^{*U}(\mu(s^t))} - \kappa^* l^{*U'}(\mu(s^t))}{\frac{C^{U'}(\mu(s^t))}{C^U(\mu(s^t))} - \kappa l^{U'}(\mu(s^t))} = \lambda(s^t)
\end{aligned}
$$

Plugging in consumption and labor as a function of the monetary stance, one arrives at

$$
\begin{aligned}
& -\left[\frac{1}{\mu(s^t)} - \frac{(1-\gamma)a^{-1}(s_t)}{\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{-1}(r)\mu_r^U} - \frac{\gamma a^{*-1}(s_t)}{\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{*-1}(r)\mu_r^U}\right.\\
& \left. - \Theta^{*U}\left(\frac{a^{*-1}(s_t)}{\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{*-1}(r)\mu_r^U} - \frac{a^{*-1}(s_t)\mu(s^t)a_r^{*-1}(s_t)}{(\sum_{r=1}^S p_s r a^{*-1}(r)\mu_r^U)^2}\right)\right]\\
& \left[\frac{1}{\mu(s^t)} - \frac{\gamma a^{-1}(s_t)}{\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{-1}(r)\mu_r^U} - \frac{(1-\gamma)a^{*-1}(s_t)}{\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{*-1}(r)\mu_r^U}\right.\\
& \left. - \Theta^{U}\left(\frac{a^{-1}(s_t)}{\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{-1}(r)\mu_r^U} - \frac{a^{-1}(s_t)\mu(s^t)a^{-1}(s_t)}{(\sum_{r=1}^S p(s^{t+1}\,|\,s^t)a^{-1}(r)\mu_r^U)^2}\right)\right]^{-1} = \lambda(s^t)
\end{aligned}
$$

If monetary policy announces not to consider employment in their objective function to avoid any inflationary bias, the optimal rule is

$$-\left[\frac{1}{\mu(s^t)}-\frac{(1-\gamma)a^{-1}(s_t)}{\sum_{r=1}^{S}p(s^{t+1}\,|\,s^t)a^{-1}(r)\mu_r^U}-\frac{\gamma a^{*-1}(s_t)}{\sum_{r=1}^{S}p(s^{t+1}\,|\,s^t)a^{*-1}(r)\mu_r^U}\right].$$
$$\left[\frac{1}{\mu(s^t)}-\frac{\gamma a^{-1}(s_t)}{\sum_{r=1}^{S}p(s^{t+1}\,|\,s^t)a^{-1}(r)\mu_r^U}-\frac{(1-\gamma)a^{*-1}(s_t)}{\sum_{r=1}^{S}p(s^{t+1}\,|\,s^t)a^{*-1}(r)\mu_r^U}\right]^{-1}=\lambda(s^t)$$

Taking the derivative with respect to $U(s^{t+1})$ into account give

$$+\beta p(s^{t+1}\,|\,s^t)V'(s^{t+1},U(s^{t+1}))+\lambda(s^t)\beta p(s^{t+1}\,|\,s^t)+\beta p(s^{t+1}\,|\,s^t)\phi_{sr}+\beta p(s^{t+1}\,|\,s^t)\zeta(s^{t+1})V'(s^{t+1},U(s^{t+1}))$$
$$\frac{\lambda(s^t)+\phi(s^{t+1})}{1+\zeta(s^{t+1})}=-V_r'(U(s^{t+1}))$$

The envelope condition gives $\lambda(s^t)=-V_s'(U(s^t))$. It states that the relative weight today $\lambda(s^t)$ (the Lagrange multiplier) is equal to the Marginal rate of transformation of the social planner. This transformation states how much marginal utility loss occurs for F when marginal utility for H is increased marginally. Linking these three conditions together with the complementary slackness conditions (this condition shows when a constraint is binding or not) gives an equation that described the evolution for the relative weight $\lambda(s^t)$:

$$\lambda(s^{t+1})\begin{cases}=\underline{\lambda}(s^{t+1}) & \text{if } \lambda(s^t)<\underline{\lambda}(s^{t+1})\\ =\lambda(s^t) & \text{if } \lambda(s^t)\in[\underline{\lambda}(s^{t+1}),\bar{\lambda}(s^{t+1})]\\ =\bar{\lambda}(s^{t+1}) & \text{if } \lambda(s^t)>\bar{\lambda}(s^{t+1}).\end{cases}$$

Note the following: The path of $\lambda(s^t)$ should be the same for both policy instruments. The only difference is how the ratio is achieved.

A.15 Figures

The following graph depicts the Pareto frontier for the initial state bb (boom in both countries).

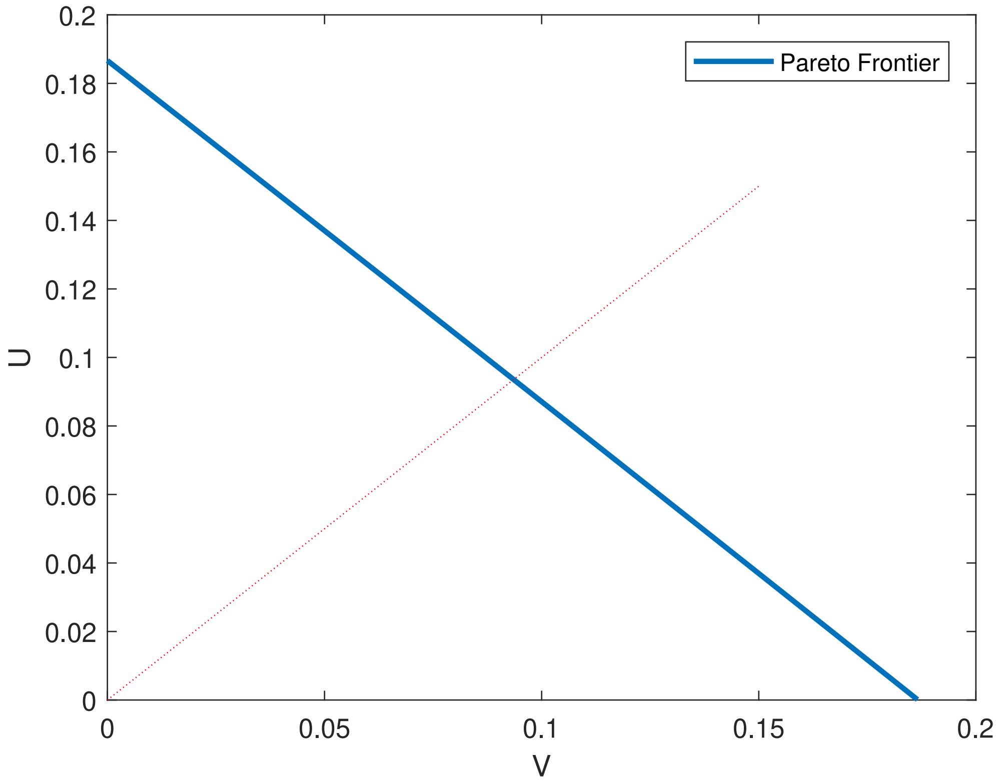


Figure 11: Pareto frontier, when both countries are initially in a boom. The red dashed line is the 45-degree line.

The Pareto frontier is indeed concave. If V is zero U reaches its maximum value, meaning that all the gains go to country H.

Terms of trade:

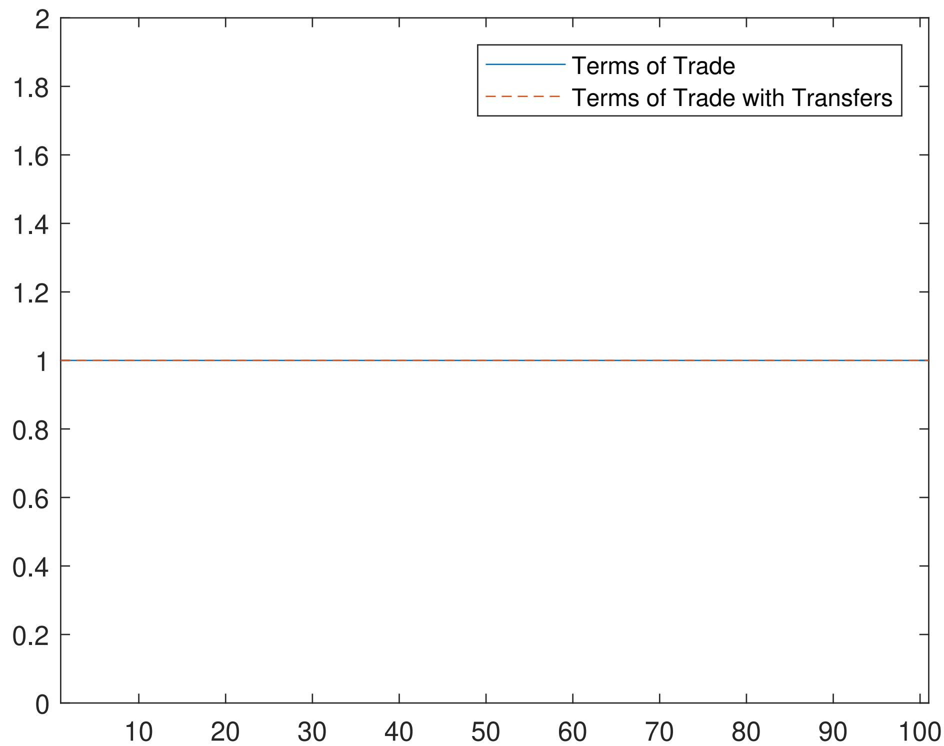


Figure 12: Evolution of terms of trade over time with and without transfers.

As soon as transfers are in place in period 18, terms of trade permanently shift upwards.

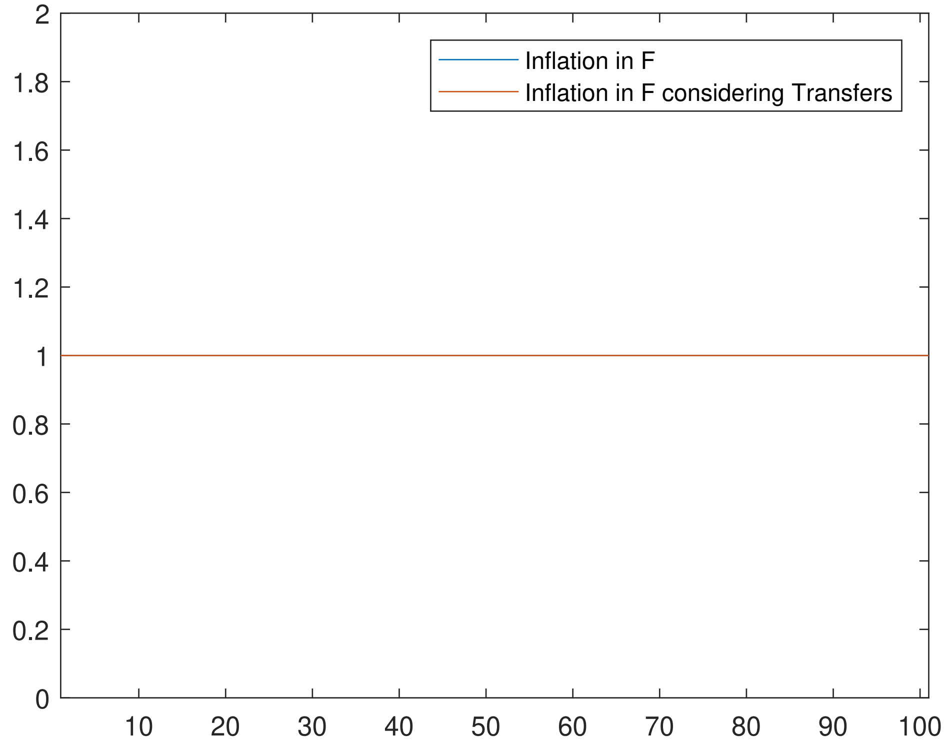


Figure 13: Evolution of Inflation over time with and without transfers.

Inflation is the same, except for the period in which transfers are announced. Only minor jump in period, not visible here.

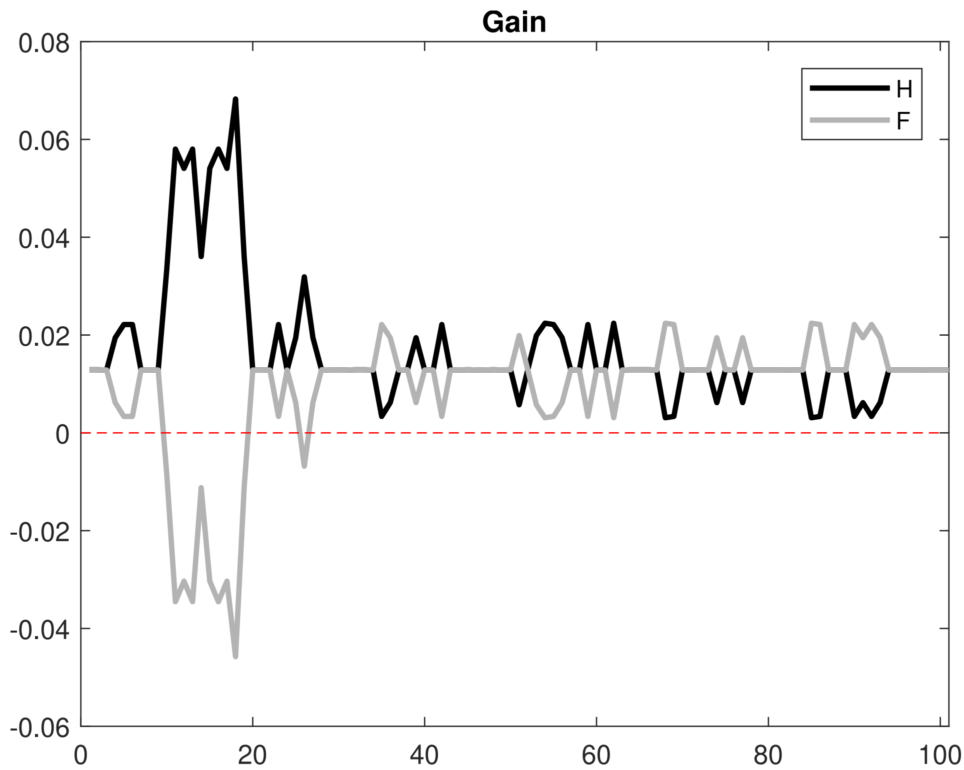


Figure 14: Evolution of gains without transfers.

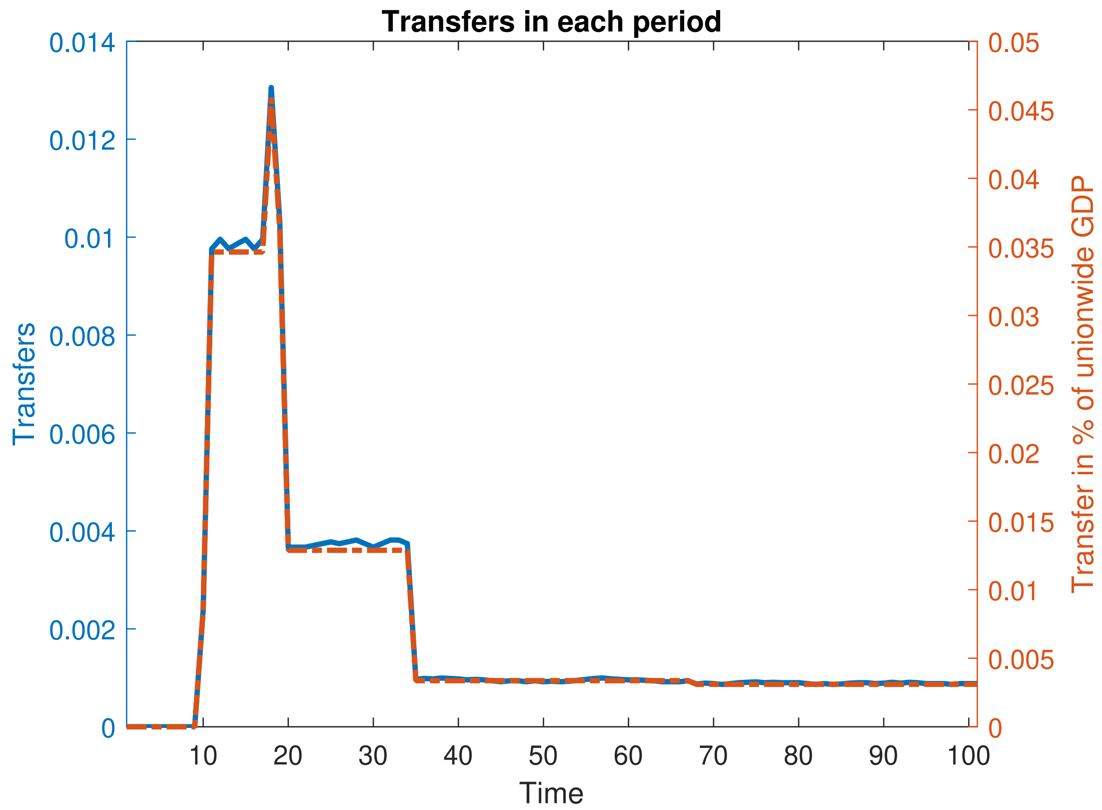


Figure 15: Evolution of transfers.

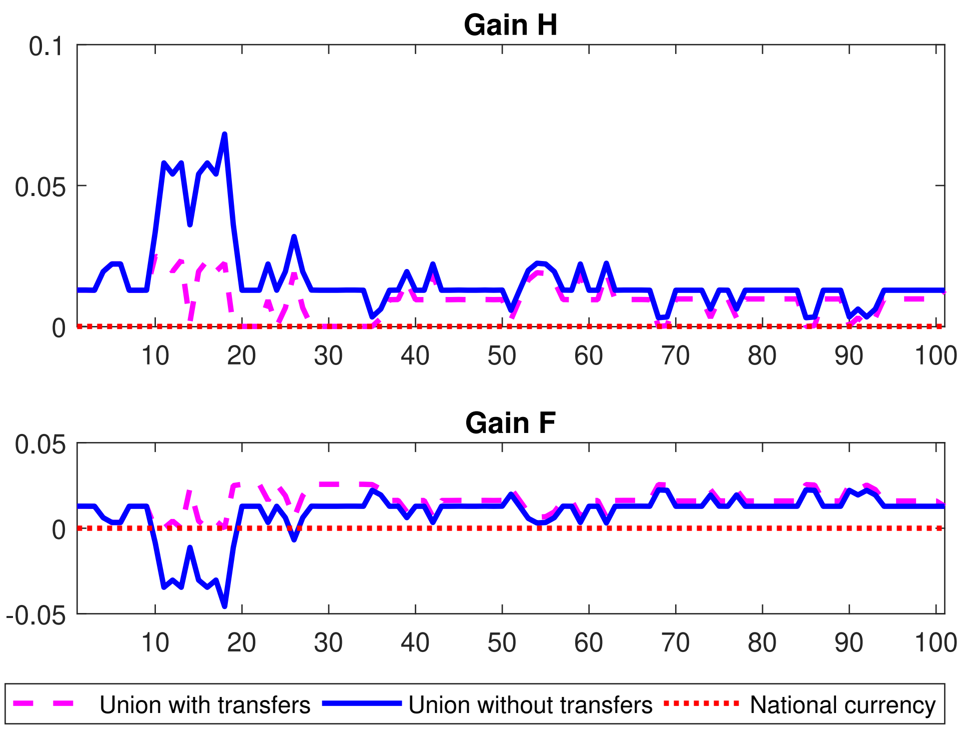
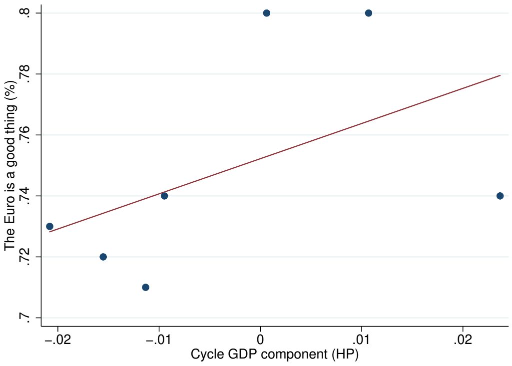


Figure 16: Evolution of gains with and without transfers.

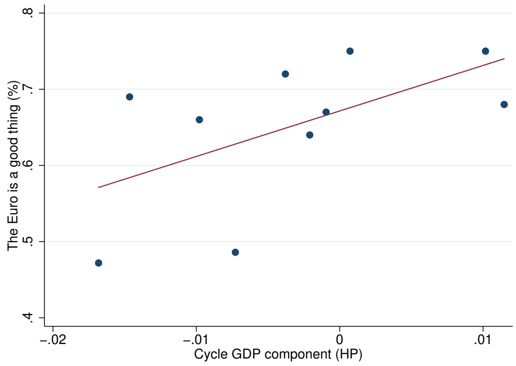
Empirical evidence for recession countries leaving the union

Figure 17: Cyclical HP GDP component and Eurobarometer: Is the Euro a good thing?

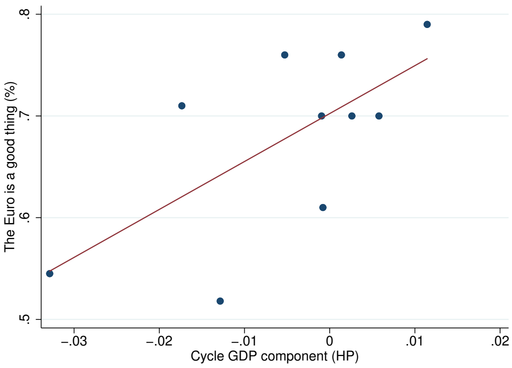
(a) Spain



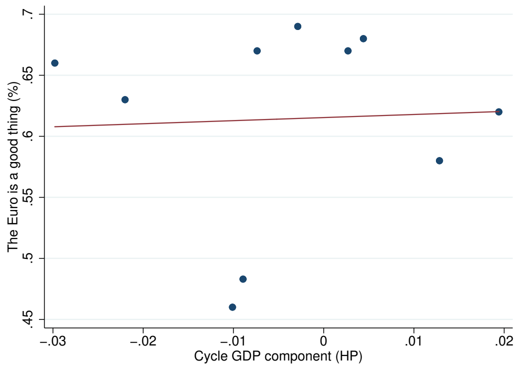
(b) France



(c) Germany



(d) Italy



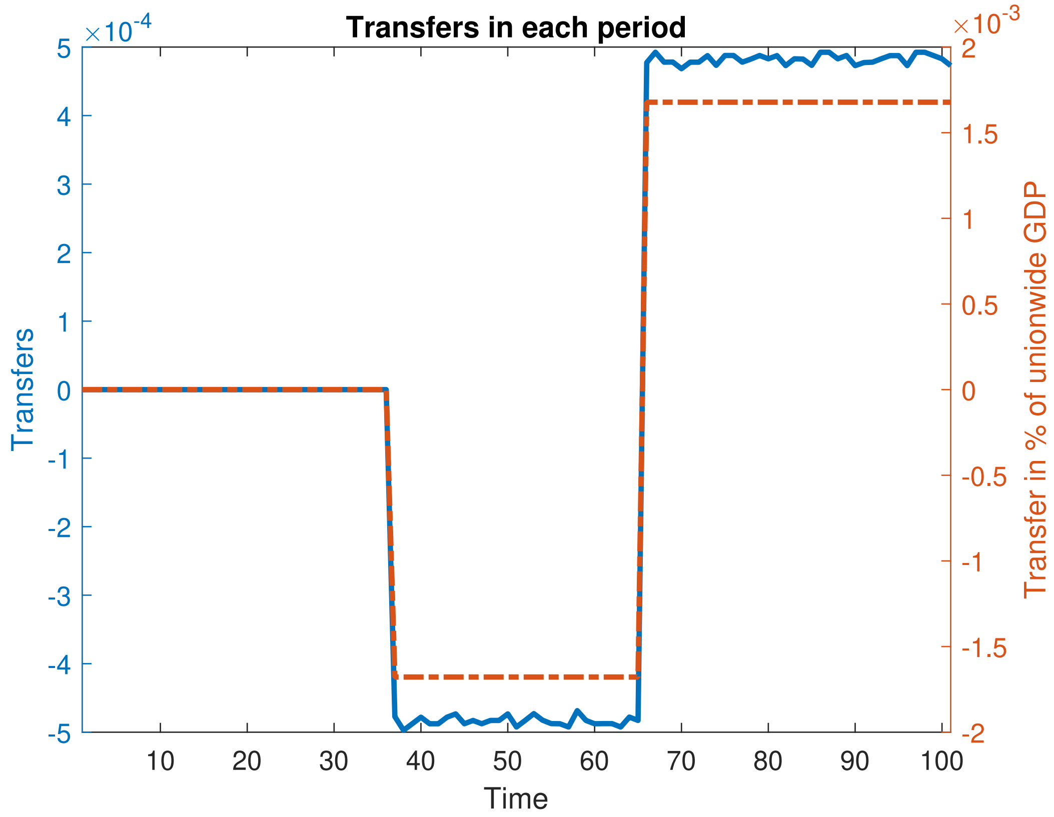


Figure 18: Evolution of transfers with one-time monetary intervention, trade costs reduction of 6.5%.

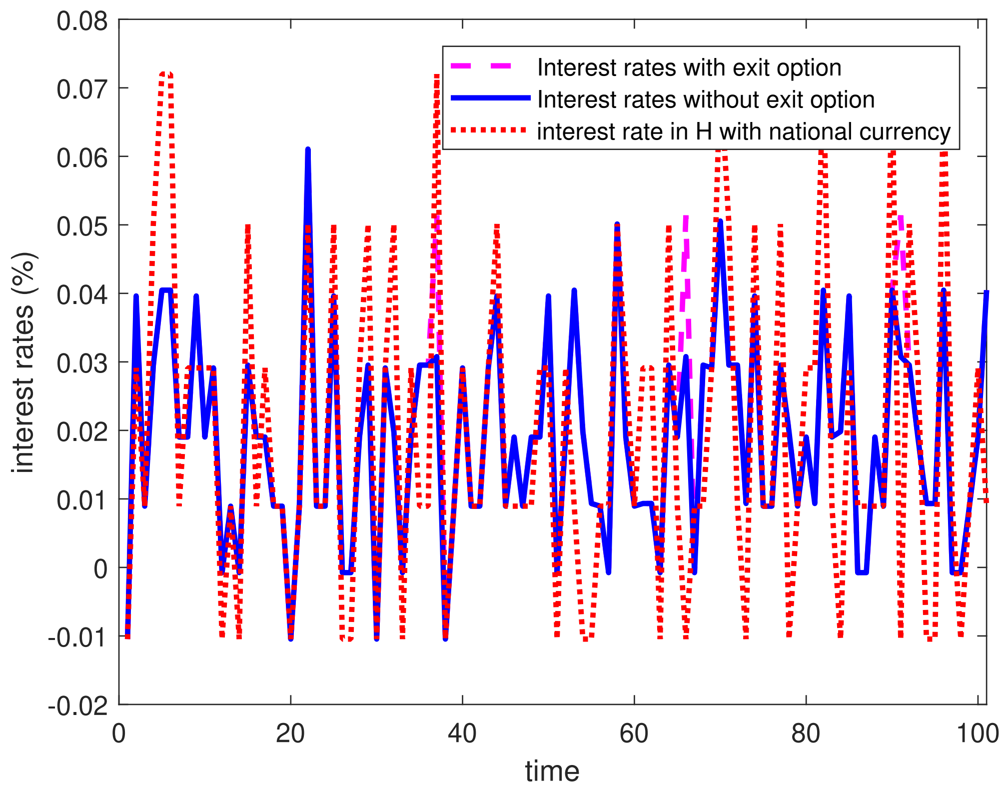


Figure 19: Evolution of interest rates with one-time monetary intervention, trade costs reduction of 6.5%

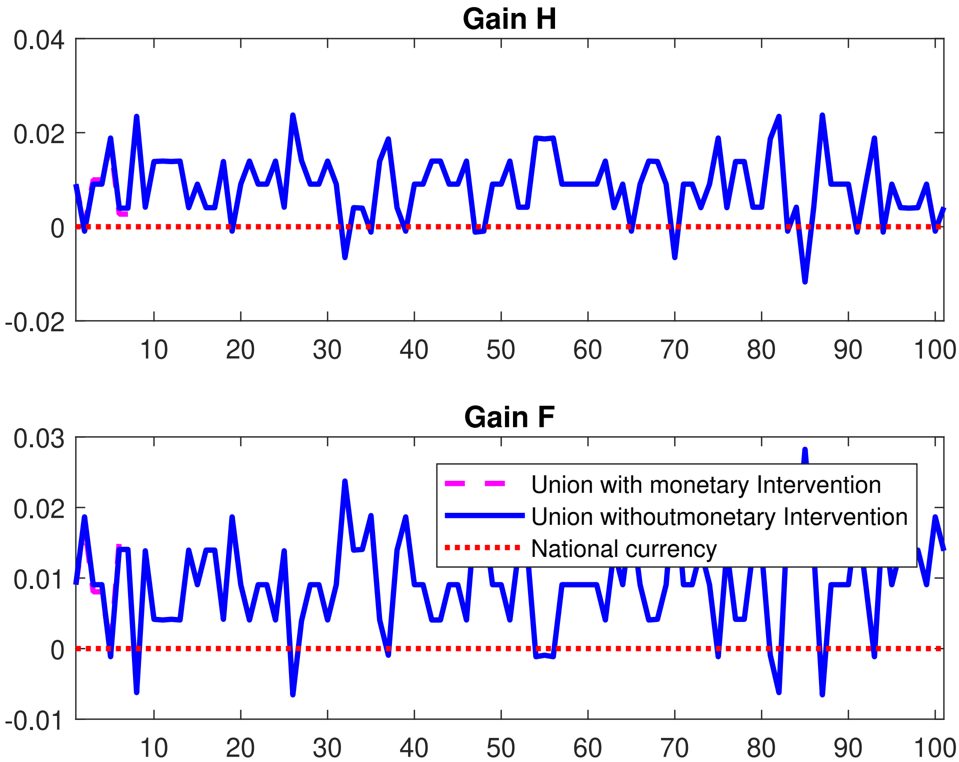


Figure 20: Evolution of gains with union-wide central bank only, trade costs 5%

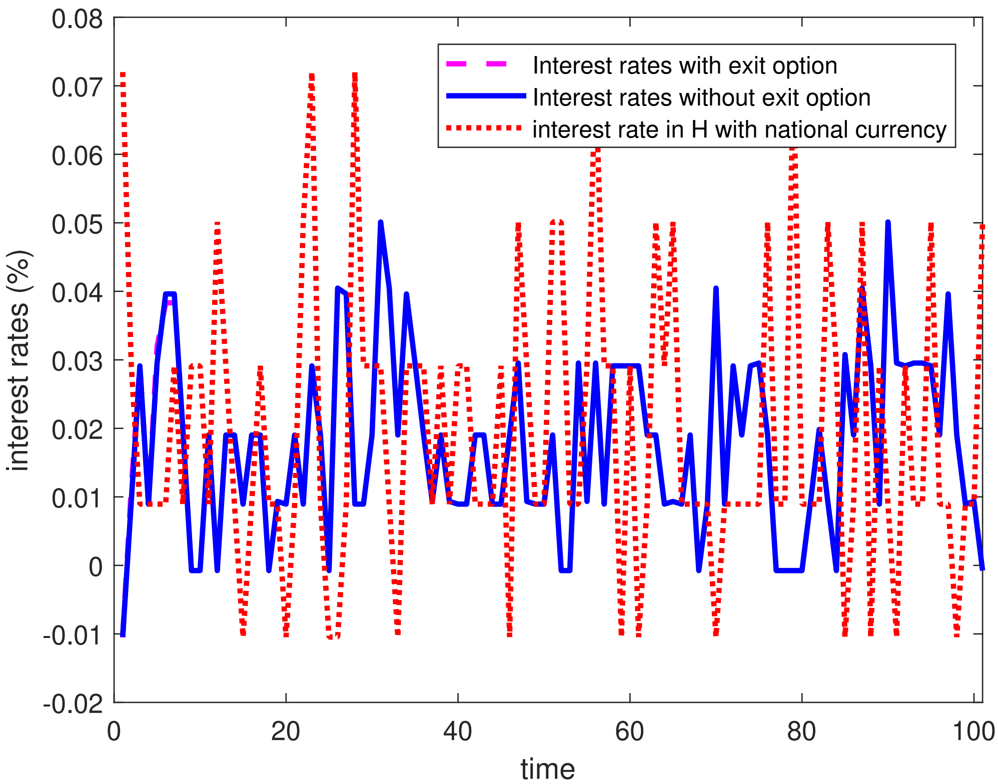


Figure 21: Evolution of interest rates with a trade cost reduction of 5% and a permanent union-wide central bank intervention.

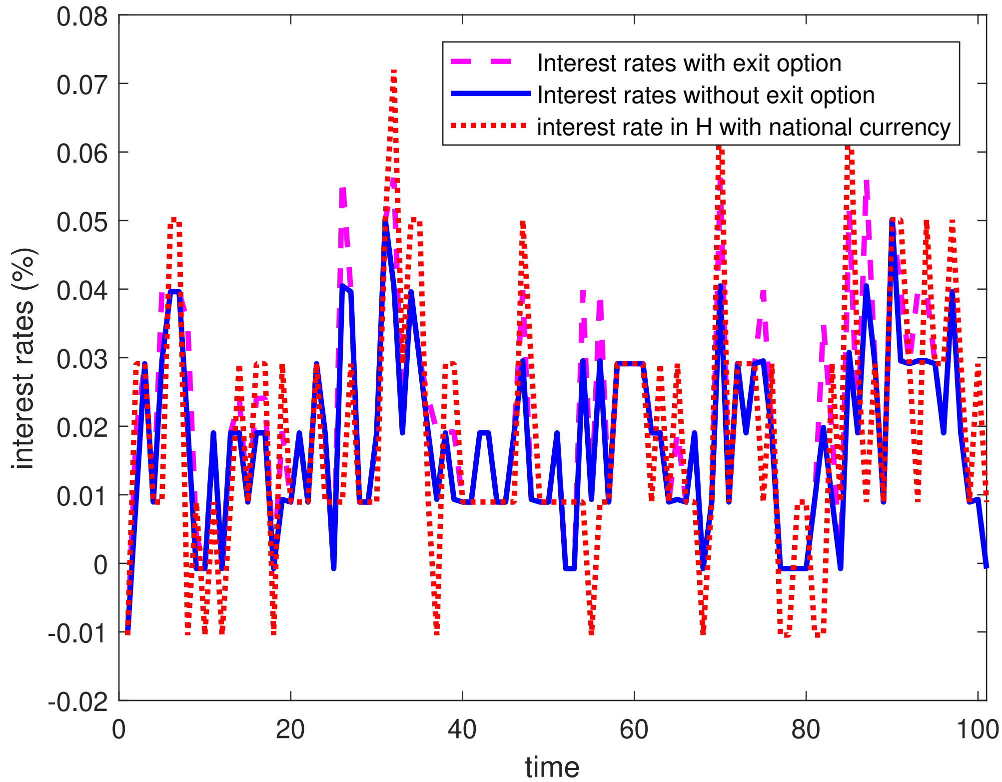


Figure 22: Evolution of interest rates with a trade cost reduction of 5% and a one-time monetary intervention.

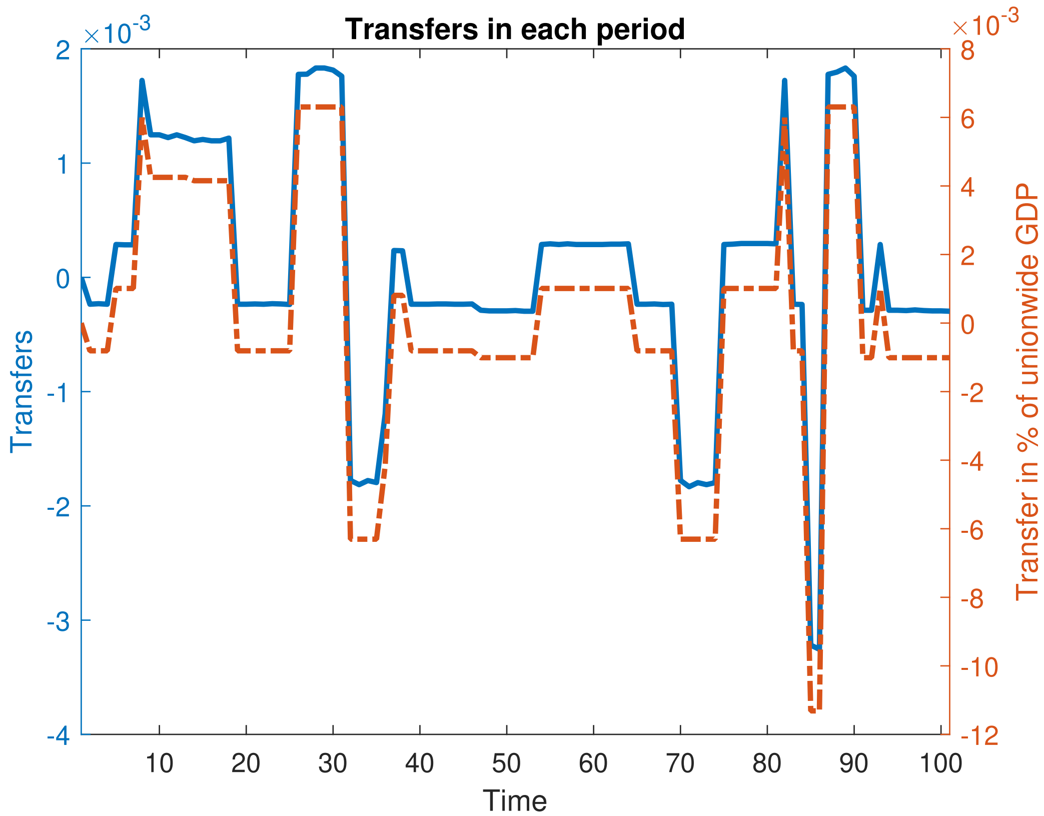


Figure 23: Evolution of transfers with one-time monetary intervention, trade cost reduction of 5%.